

# Starting Points for Designing Freeform Four-Mirror Telescopes

Jonathan Papa

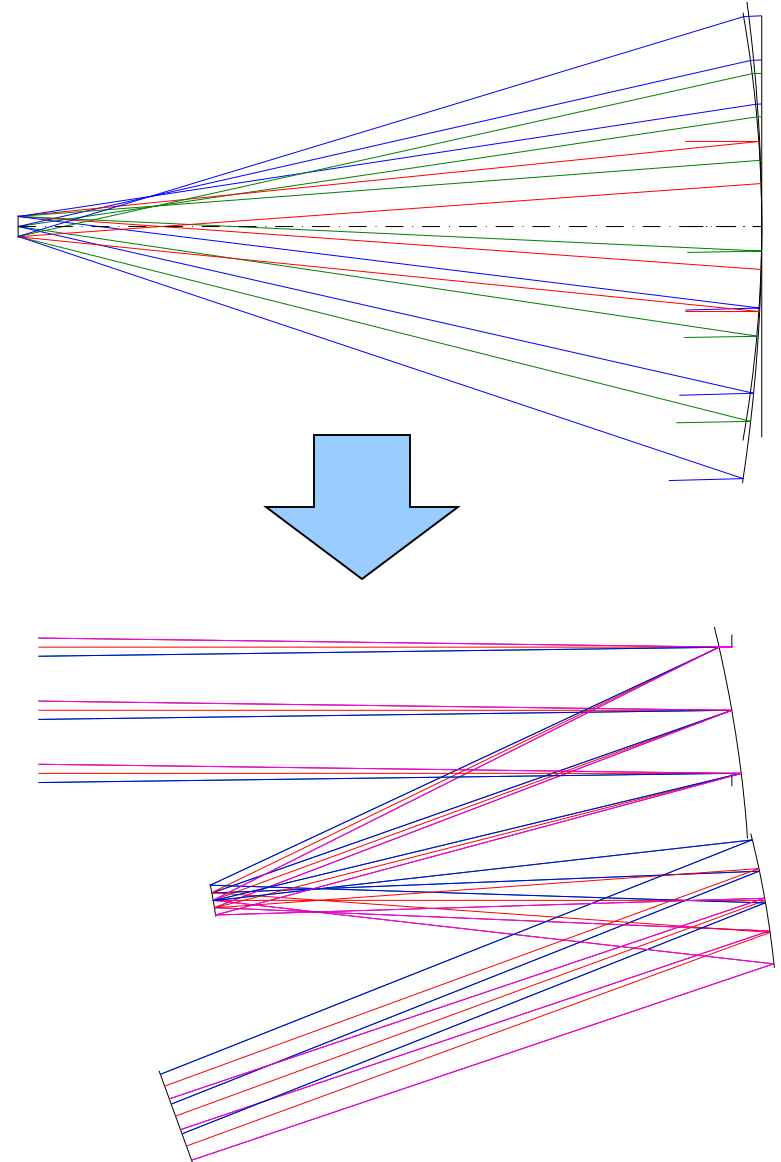
Jannick Rolland (PI)

Joseph Howard (NASA mentor)

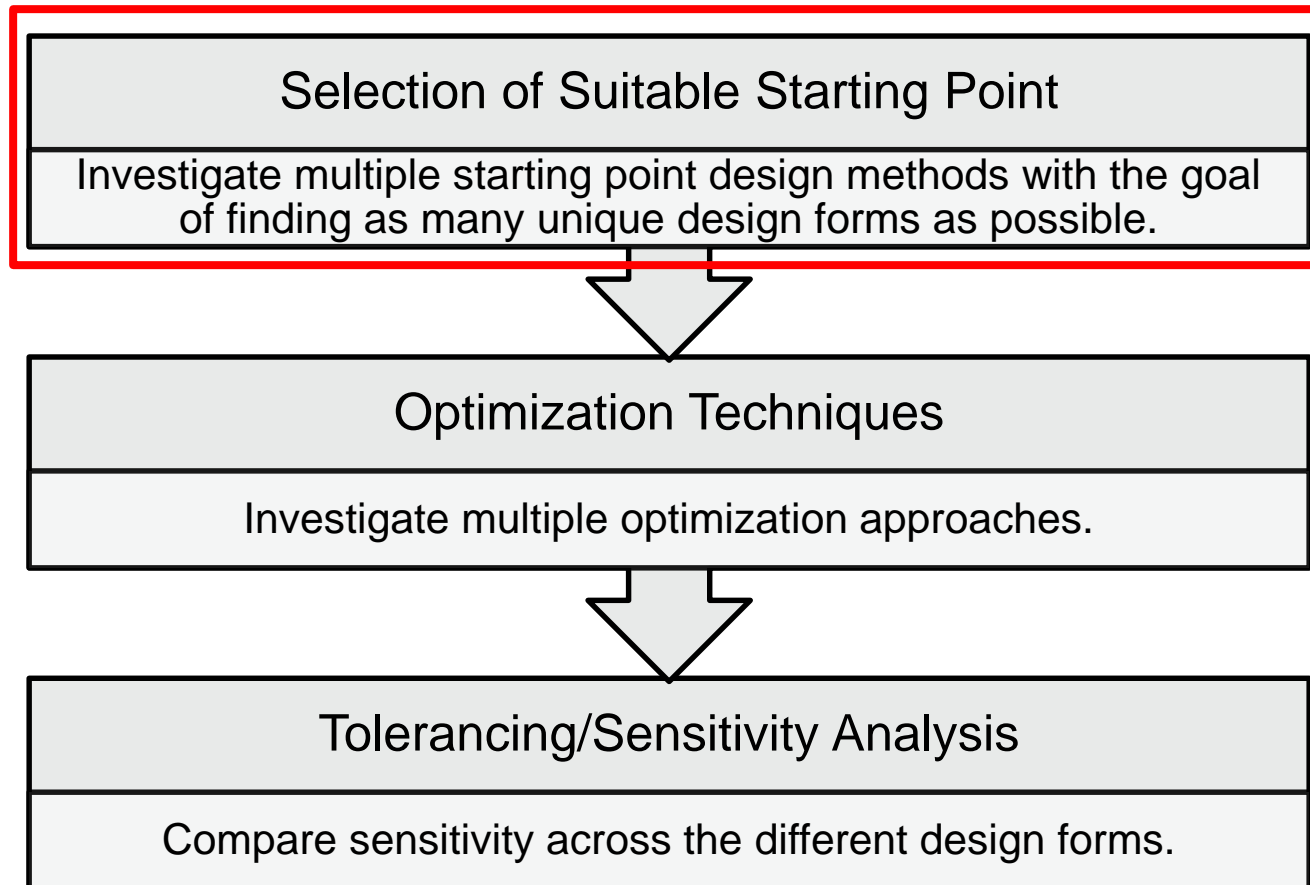


# Project Overview

- Central Objective: Survey of the four-mirror freeform solution space that considers geometries that could be advantageous for system constraints, such as mass, volume, stray light control, or radiation shielding.
- Methods/Techniques: Use analytically designed starting points before adding freeform terms to explore different design forms.



# Parts of the Design Process



# Considered Starting Points So Far (This is not an exhaustive list)

Method	Corrected through 3 <sup>rd</sup> Order	Stigmatic Imaging at Every Surface (One Field Point)	Unobscured
Rotationally-Symmetric Rakich All-Spherical Maps	●		
Rotationally-Symmetric All-Conic Maps	●	●	
Off-Axis Conic Layout Tool		●	●
Off-Axis Conics from Aberration Coefficients for Plane-Symmetry	●	●	●



# Starting Points

Method	Corrected through 3 <sup>rd</sup> Order	Stigmatic Imaging at Every Surface (One Field Point)	Unobscured
Rotationally-Symmetric Rakich All-Spherical Maps	●		
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# Rotationally-Symmetric Rakich All-Spherical Maps

$$W_{spherical} = -\frac{1}{8} n^2 i_a^2 y_a \left( \frac{u_a'}{n'} - \frac{u_a}{n} \right)$$

$$\text{Total Spherical Aberration} = \sum_i W_i$$

$$\text{Total Coma} \propto \sum_i x_i W_i$$

$$\text{Total Astigmatism} \propto \sum_i x_i^2 W_i$$

**To get an anastigmat solution, the following conditions must be satisfied:**

$$W_1 + W_2 + W_3 + W_4 = 0$$

$$W_1 x_1 + W_2 x_2 + W_3 x_3 + W_4 x_4 = 0$$

$$W_1 x_1^2 + W_2 x_2^2 + W_3 x_3^2 + W_4 x_4^2 = 0$$

Rakich solves for the curvature of mirrors 3 and 4; and the thicknesses after mirrors 2, 3, and 4; as a function of the curvature of mirror 2, the thickness after mirror 1, and stop location  $x_1$ .

**Rakich, Opt. Eng. 46(10), 2007**



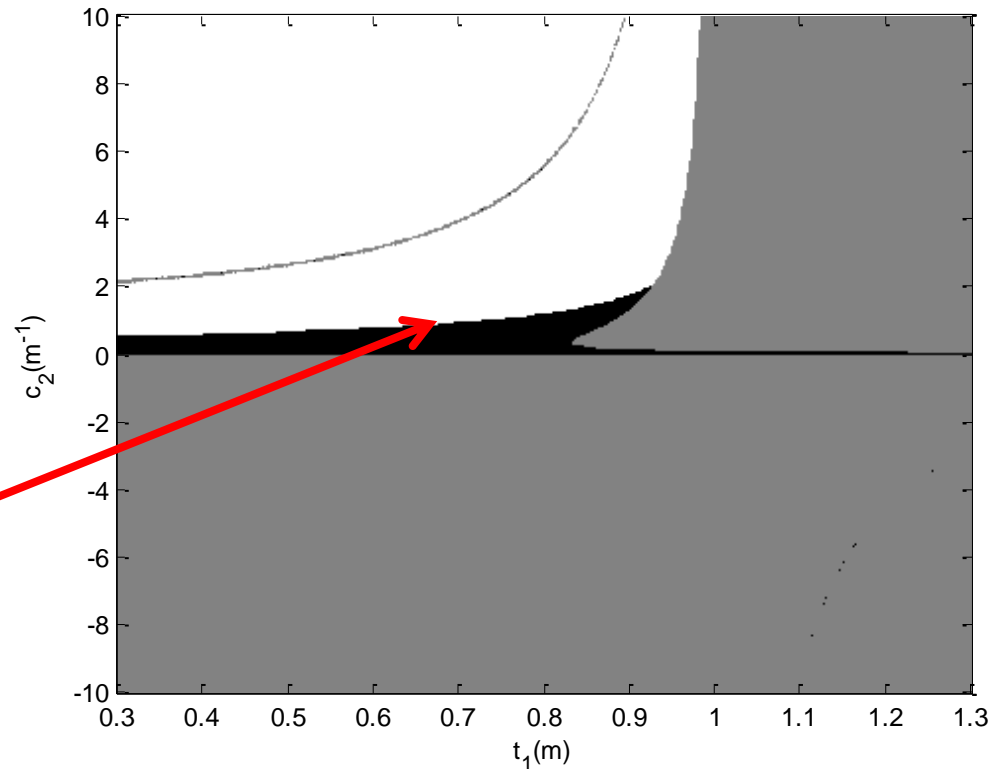
# Rotationally-Symmetric Rakich All-Spherical Maps

## Matlab Implementation:

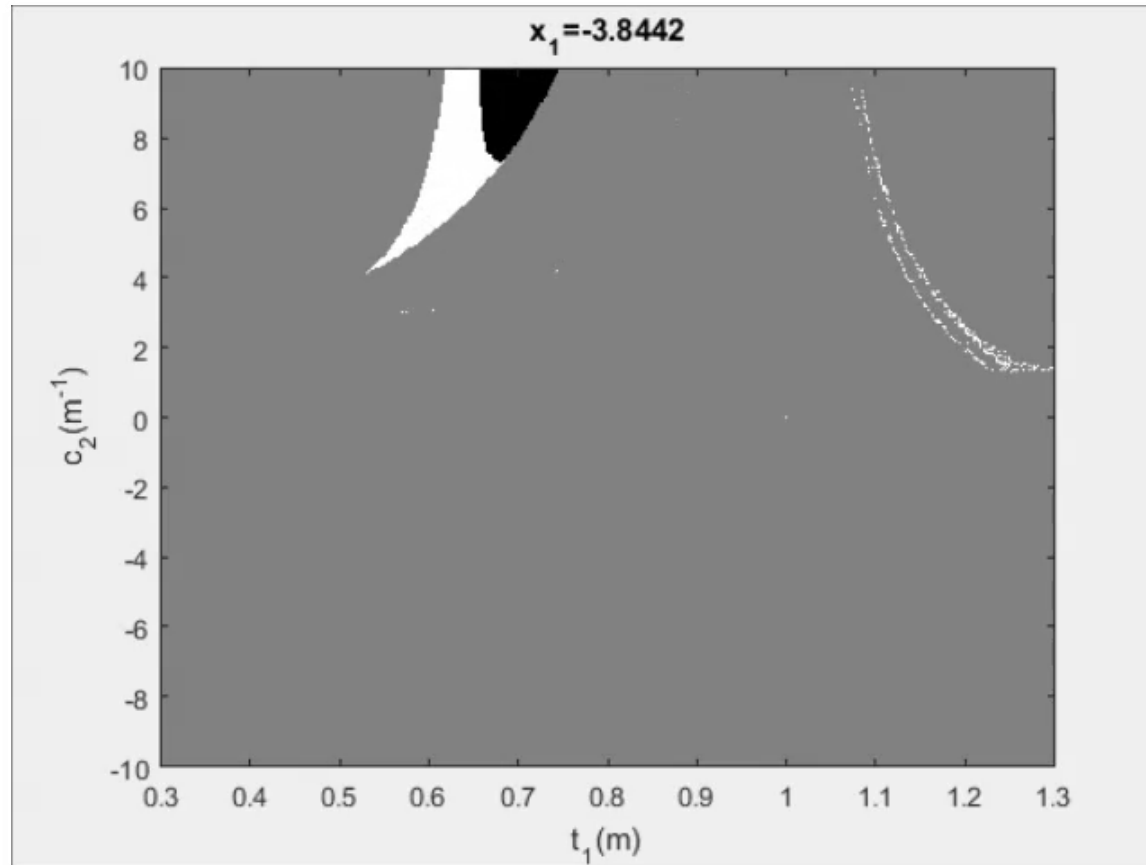
Parameters of the three dimensional solution space are  $t_1$  (thickness after mirror 1),  $c_2$  (curvature of mirror 2), and  $x_1$  (represented as time axis of video, corresponds to stop position).

Interface = Flat Field Solutions

White:  $>0$  PZT; Black:  $<0$  PZT; Gray: No Viable solution



# Video of Solution Space





# Rotationally-Symmetric Rakich All-Spherical Maps

- Pick a solution from the solution map by filtering for solutions with desirable properties; such as adequate mirror separations that allow for unobscuration by using smaller tilts, or internal images, etc.
- Unobscure by tilting the mirrors while adding freeform terms (i.e. Zernikes) through optimization.



# Validation of Solution Maps

Parameters:

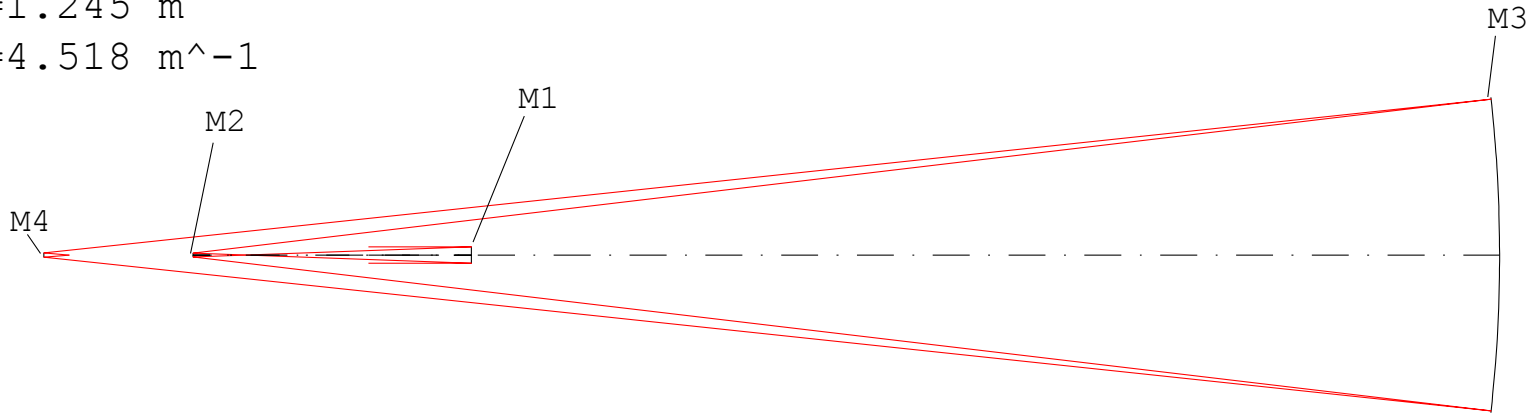
Cubic 1: solution 2

Cubic 2: solution 3

x1=2

t1=1.245 m

c2=4.518 m<sup>-1</sup>



FMA

Position 1, Wavelength = 587.6 NM

	SA	TCO	TAS	SAS	PTB	DST	AX	LAT	PTZ
STO	0.002646	0.000996	0.000083	0.000000	-0.000042	0.000000	0.000000	0.000000	0.001000
2	-0.025482	-0.015082	-0.002599	-0.000615	0.000377	-0.000121	0.000000	0.000000	-0.009036
3	0.033775	0.022323	0.004904	0.001626	-0.000013	0.000358	0.000000	0.000000	0.000320
4	-0.010939	-0.008236	-0.001972	-0.000594	0.000095	-0.000149	0.000000	0.000000	-0.002291
SUM	0.000000	0.000000	0.000417	0.000417	0.000417	0.000088	0.000000	0.000000	-0.010007

781.25 MM

FMA

Scale: 0.03 JCP 31-Mar-16



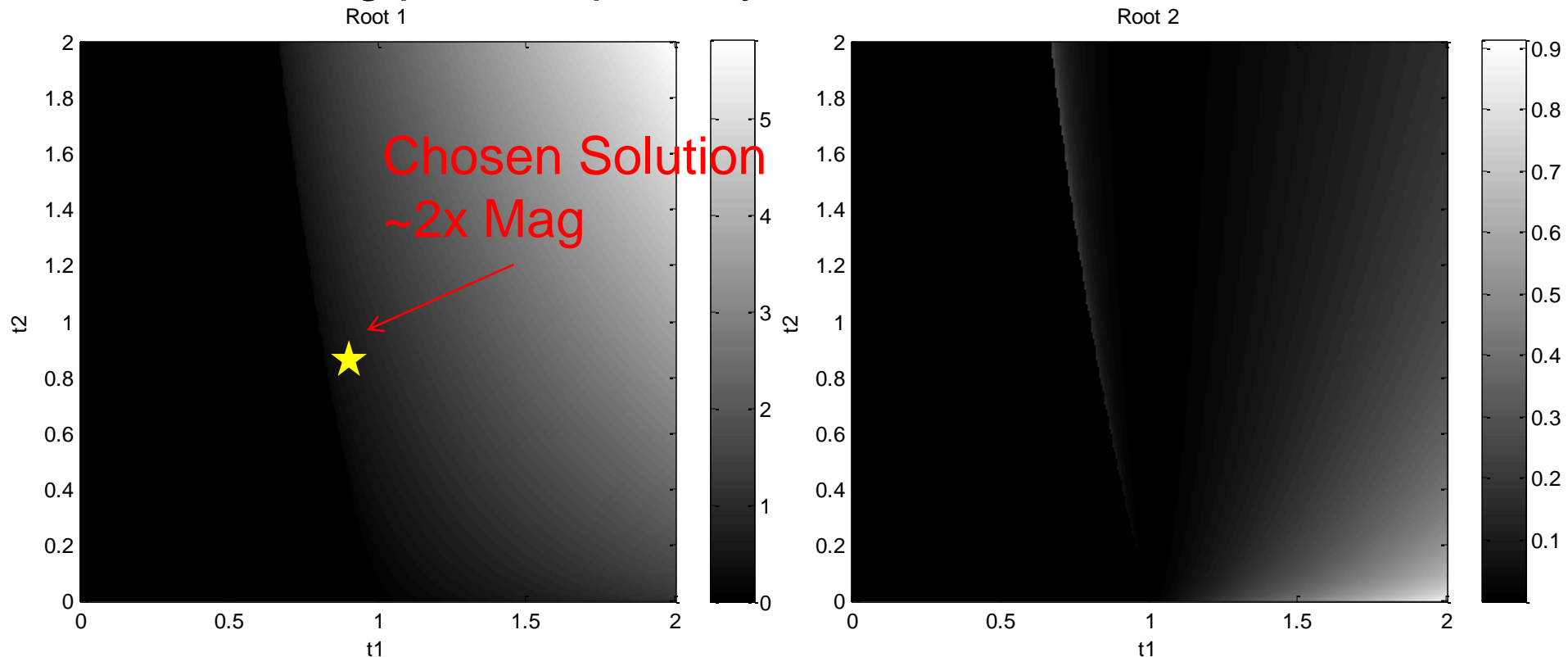
# Starting Points

Method	Corrected through 3 <sup>rd</sup> Order	Stigmatic Imaging at Every Surface (One Field Point)	Unobscured
Rotationally-Symmetric Rakich All-Spherical Maps	●		
Rotationally-Symmetric All-Conic Maps	●	●	
Off-Axis Conic Layout Tool		●	●
Off-Axis Conics from Aberration Coefficients for Plane-Symmetry	●	●	●

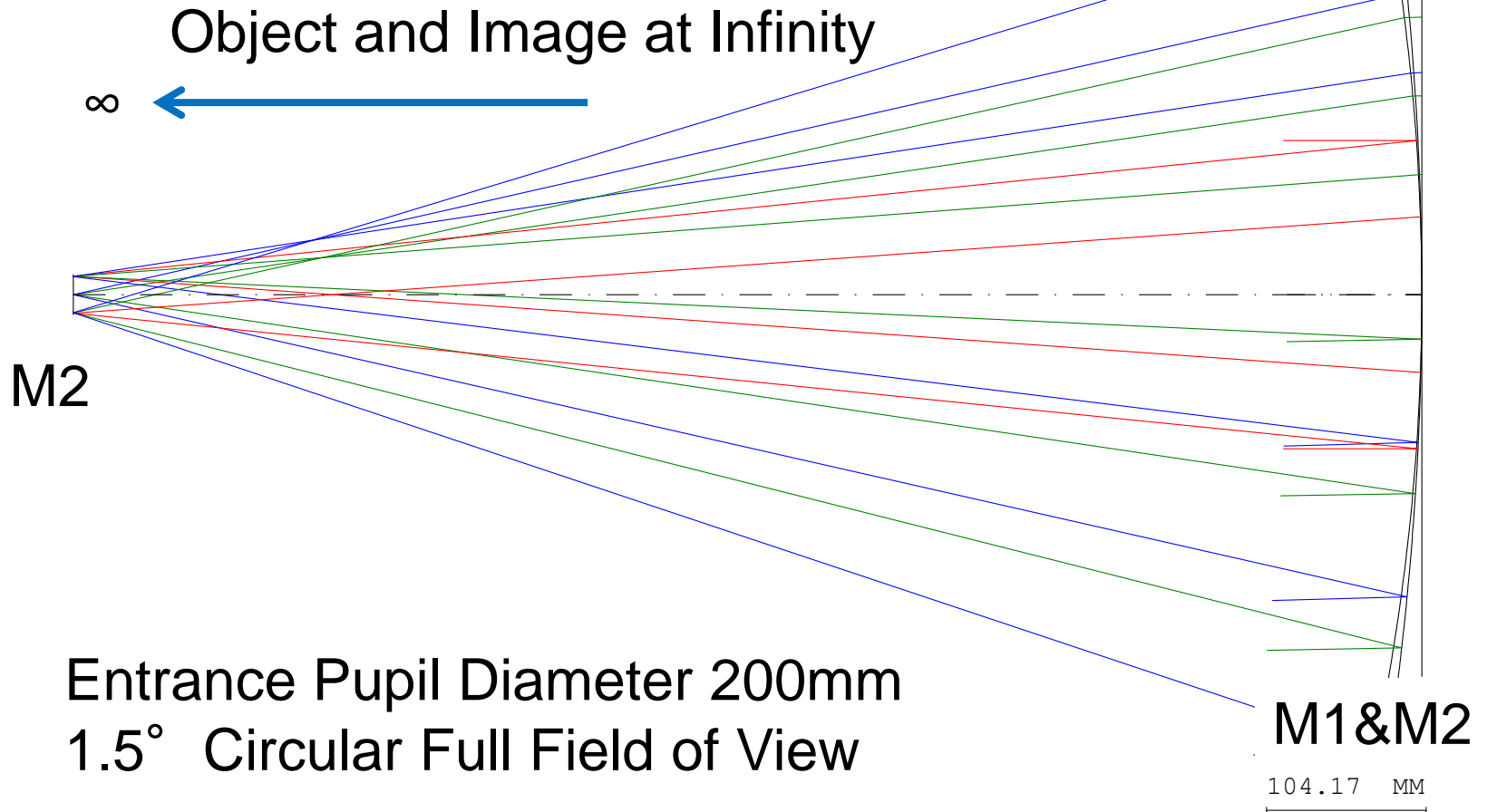


# Rotationally-Symmetric All-Conic Maps for Afocal Three Mirror Systems

- Grayscale represents magnification.
- Focal length of primary set to 1.
- Assuming positive primary.



# Selected 2x Solution from Root 1 Map; $t_1=t_2$



Afocal TMA

Scale: 0.24 JCP 22-Sep-16



# Third Order Analysis

Afocal TMA

Position 1, Wavelength = 587.6 NM

	SA	TCO	TAS	SAS	PTB	DST
1	-0.247594	1.812133	-4.353068	-1.405749	0.067910	3.429543
	0.247594	-1.423124	2.726615	0.908872		-1.741340
STO	0.025215	-0.192614	0.326974	0.000000	-0.163487	0.000000
	-0.025215	0.000000	0.000000	0.000000		0.000000
3	-0.044840	0.314107	-0.637873	-0.148907	0.095577	0.347702
	0.044840	-0.510502	1.937353	0.645784		-2.450749
SUM	0.000000	0.000000	0.000000	0.000000	0.000000	-0.414845



# Starting Points

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Rotationally-Symmetric All-Conic Maps	●	●	
Off-Axis Conic Layout Tool		●	●
Off-Axis Conics from Aberration Coefficients for Plane-Symmetry	●	●	●



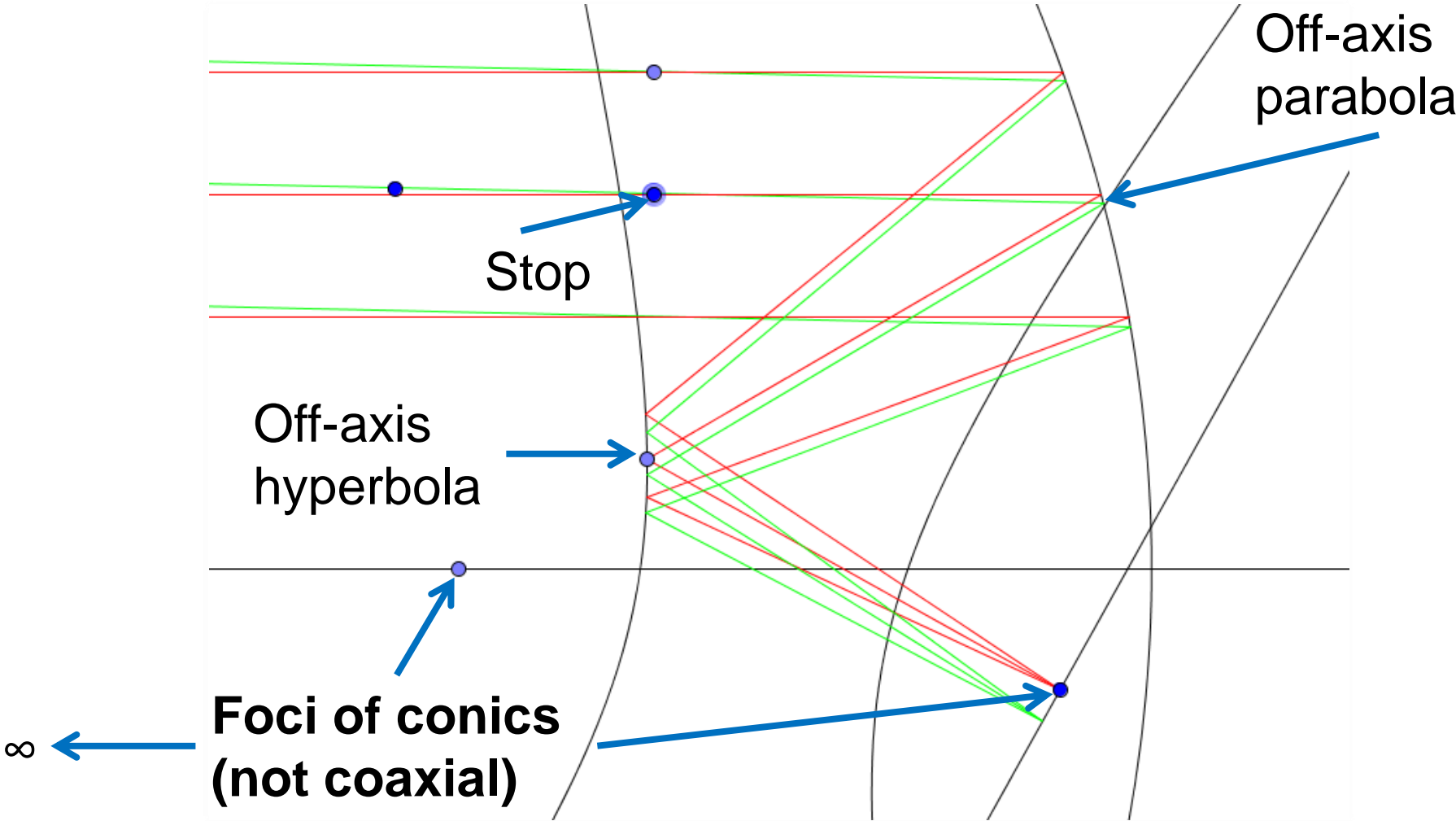
# Off-Axis Conic Layout Tool

- Force stigmatic imaging for the field point along optical axis ray (OAR)/base ray, such that all intermediate image points are stigmatic, allow “pivoting” about the foci of the conics. System is like a linkage of off-axis conic mirrors.
- When pivoting, the basal field point remains stigmatic.
- This method allows for **unobscured** starting points.

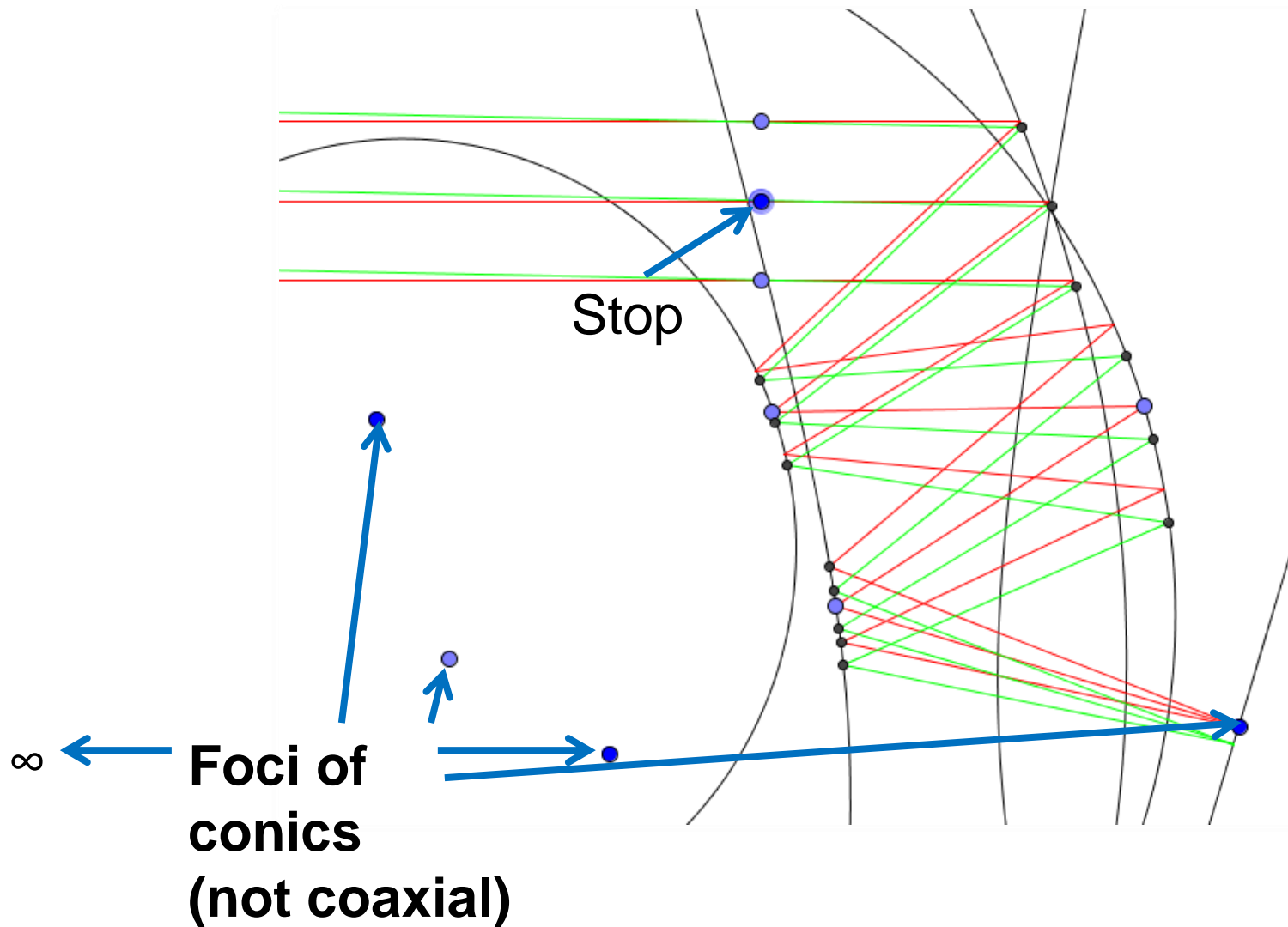




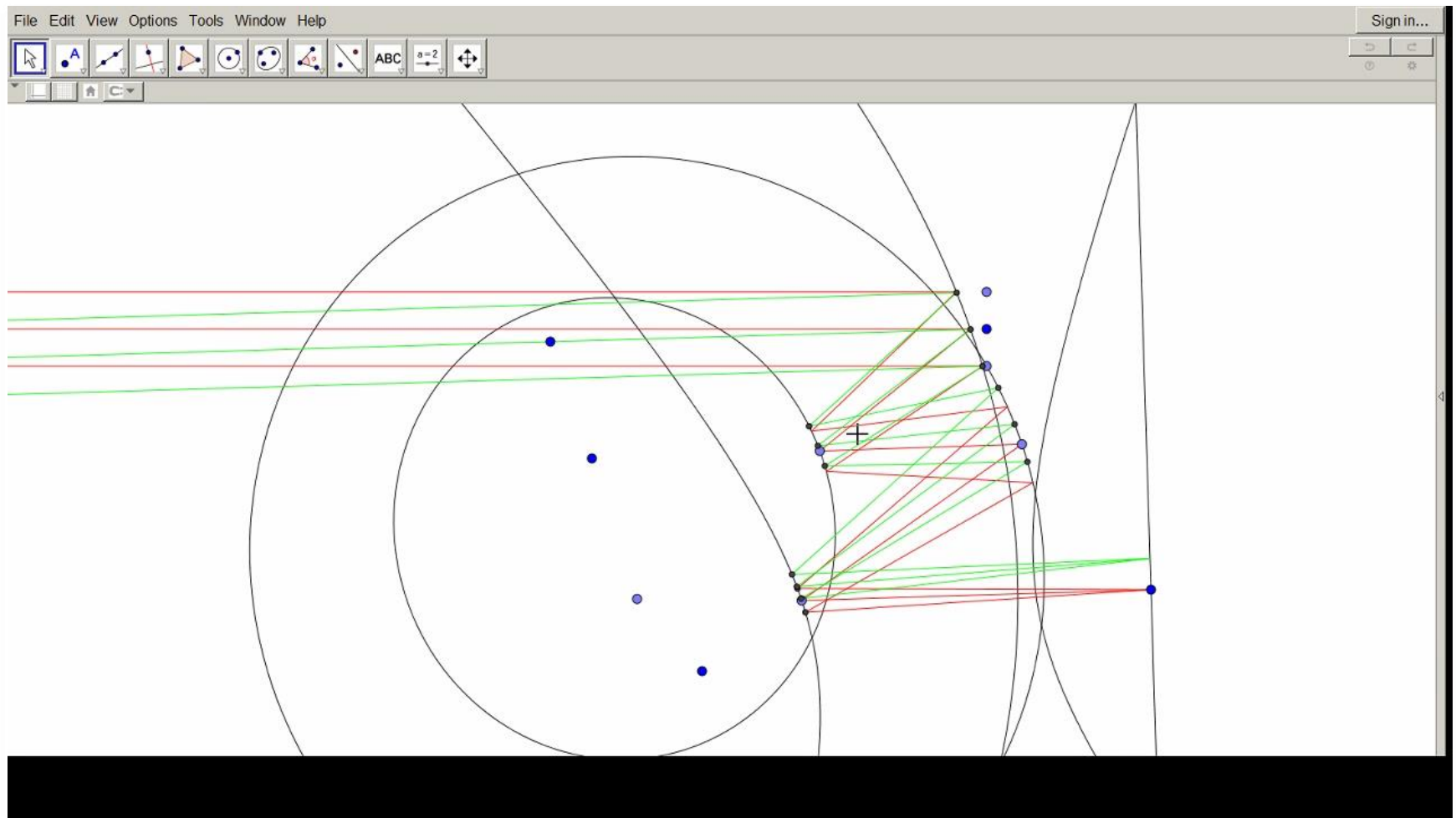
# Two Mirror Pivoting Conics



# Four Mirror Pivoting Conics



# Video of Off-Axis Conic Layout Tool



# Starting Points

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Off-Axis Conics from Aberration Coefficients for Plane-Symmetry	●	●	●



# Off-Axis Conics from Aberration Coefficients for Plane-Symmetry

- All conic foci are constrained to a plane.
- Sasian developed aberration coefficients for plane-symmetric systems that depend on paraxial raytrace quantities to third order. He demonstrated the coefficients on a two mirror system pivoting about shared conic focus.
- **This method will utilize the solutions from the “Rotationally-Symmetric All-Conic Maps” method, and take it further by unobscuring those solutions (like in the “Off-Axis Conic Layout Tool”, but this method is corrected through 3<sup>rd</sup> order instead of just at one field point).**

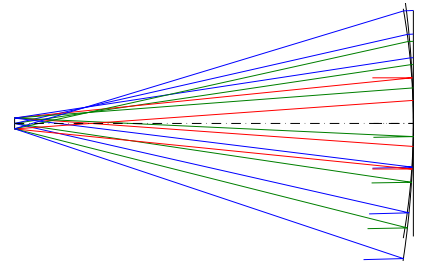
*Sasian, Opt. Eng. 33(6), 1994*



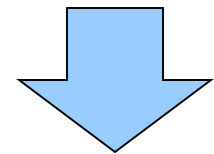
# Aberration Coefficients Before and After Tilting/Unobscuring

In waves zero-to-peak at 587.5618 nm; before tilting

Aberrations	Surface 1	Surface 2	Surface 3	Sum
Linear Coma	-5.57	2.76	2.81	0.00
Field Lin. Field Asym. Ast.	0.00	0.00	0.00	0.00
Quadratic Astigmatism	1.46	1.77	-3.23	0.00
Field Curvature	0.00	2.64	-2.64	0.00

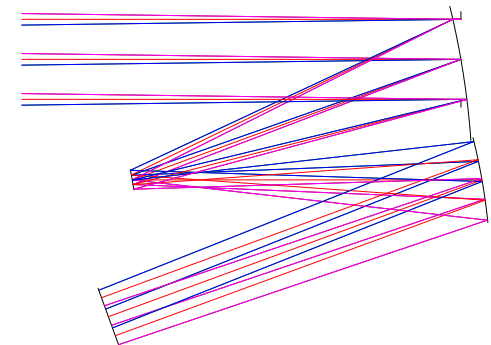


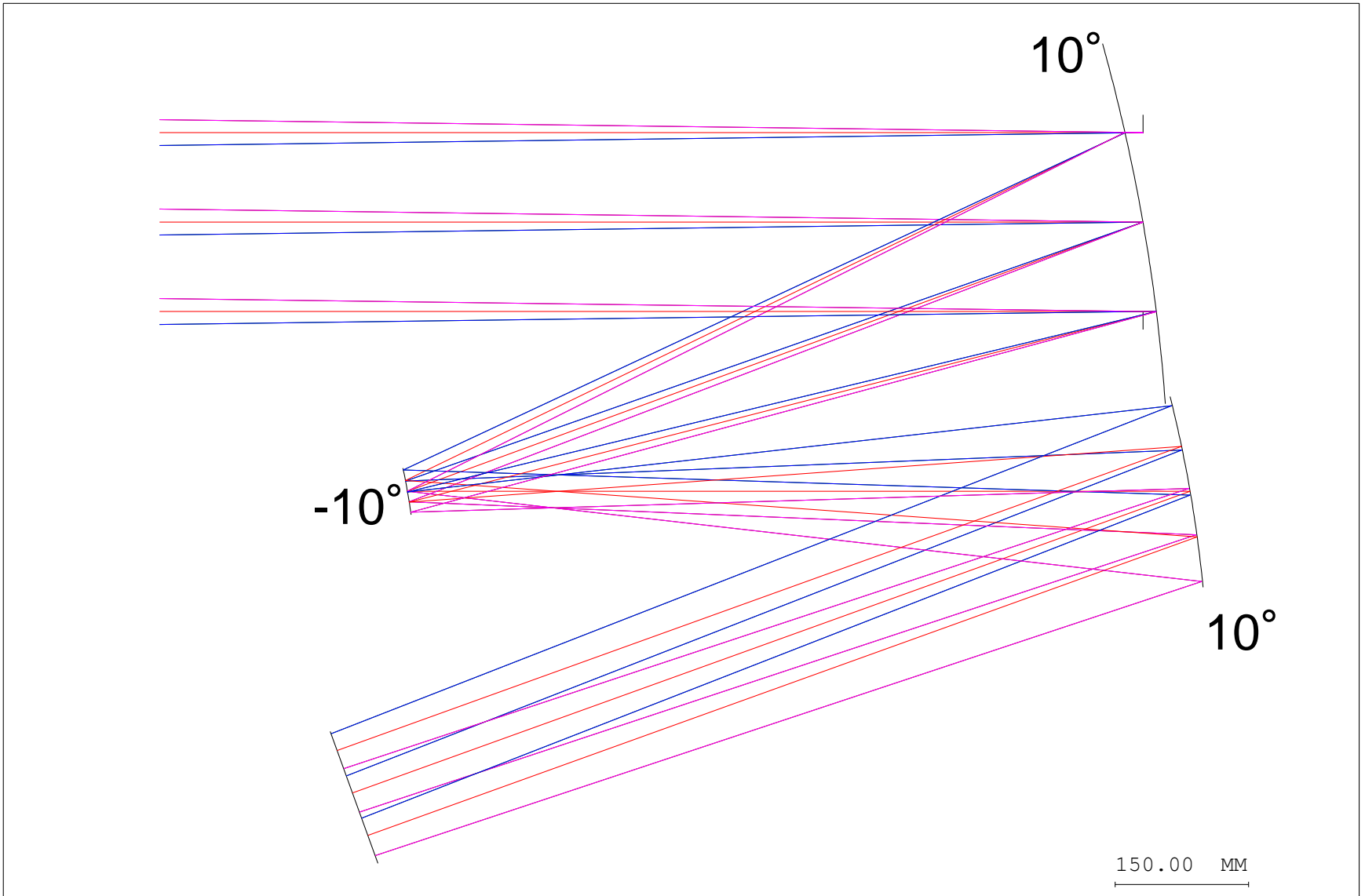
Tilt mirrors to unobscure while canceling introduced linear astigmatism, as we tilt.



	Surface 1	Surface 2	Surface 3
Tilt	10°	-10°	10°

Aberrations	Surface 1	Surface 2	Surface 3	Sum
Linear Coma	-5.49	2.76	2.85	0.13
Field Lin. Field Asym. Ast.	38.69	-11.20	-27.49	0.00
Quadratic Astigmatism	1.46	1.79	-3.31	-0.07
Field Curvature	0.01	2.62	-2.67	-0.03



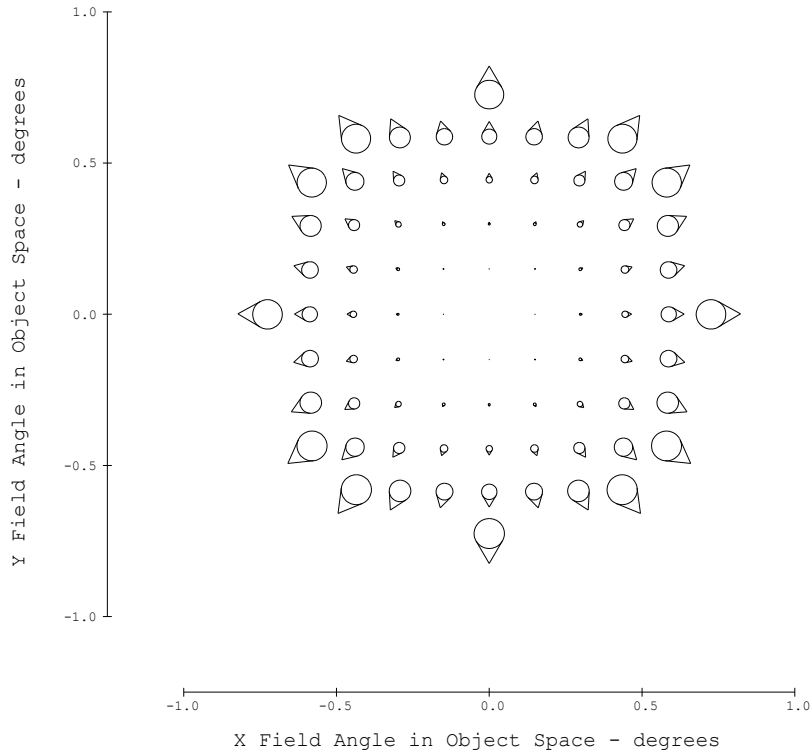


Afocal TMA

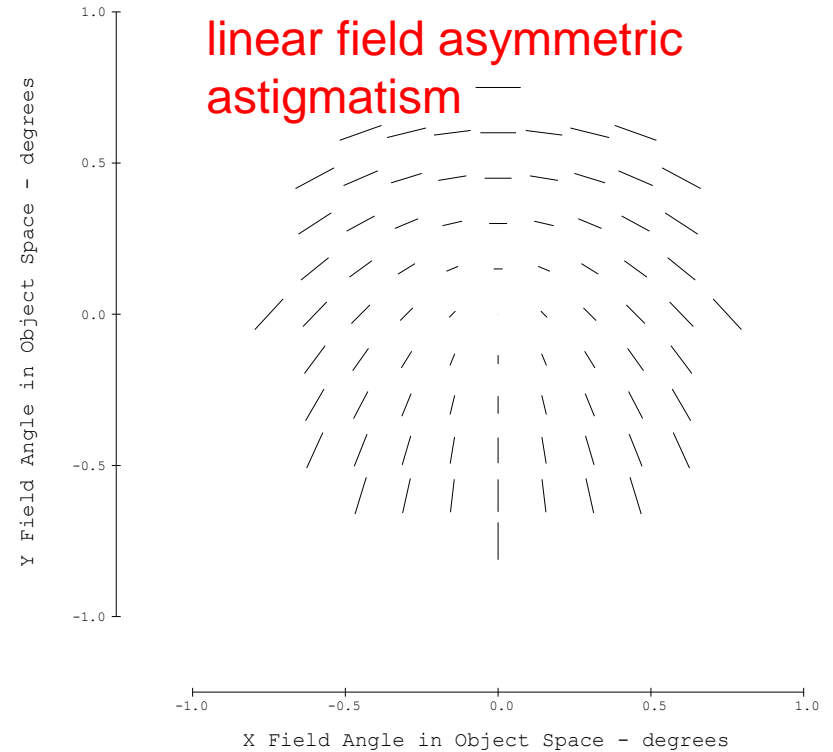
JCP 22-Sep-16



# Full Field Displays (Real Raytracing)



There is some residual field  
linear field asymmetric  
astigmatism



Afocal TMA

FRINGE ZERNIKE PAIR Z7 AND Z8  
vs  
FIELD ANGLE IN OBJECT SPACE

Minimum = 0.59538e-8  
Maximum = 0.0050796  
Average = 0.0021227  
Std Dev = 0.0016161

Afocal TMA

FRINGE ZERNIKE PAIR Z5 AND Z6  
vs  
FIELD ANGLE IN OBJECT SPACE

Minimum = 0.26505e-7  
Maximum = 0.21615  
Average = 0.13991  
Std Dev = 0.047951

JCP

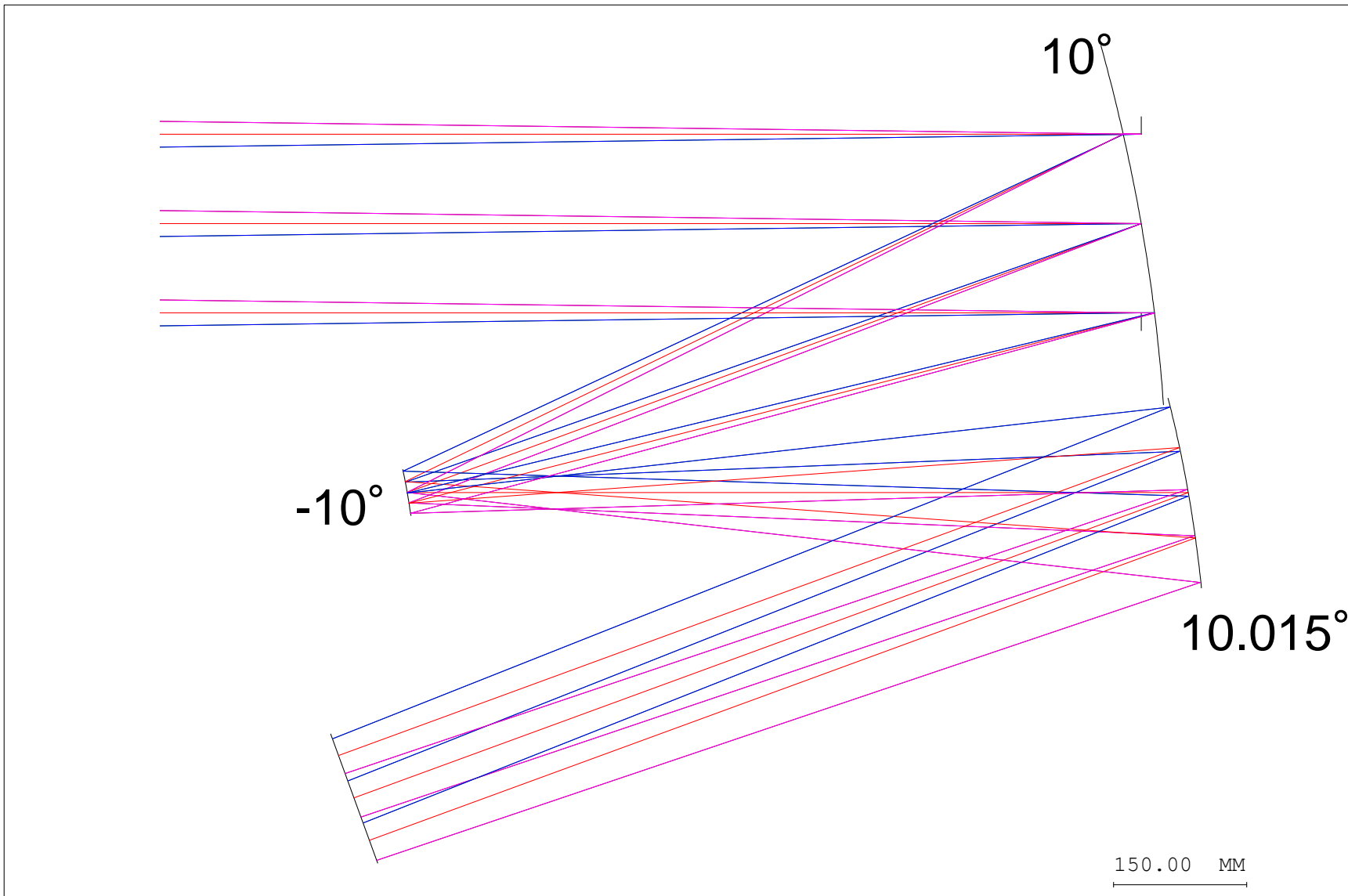
22-Sep-16

Worst field point has RMSWE 0.09λ

0.6waves ( 587.6 nm)



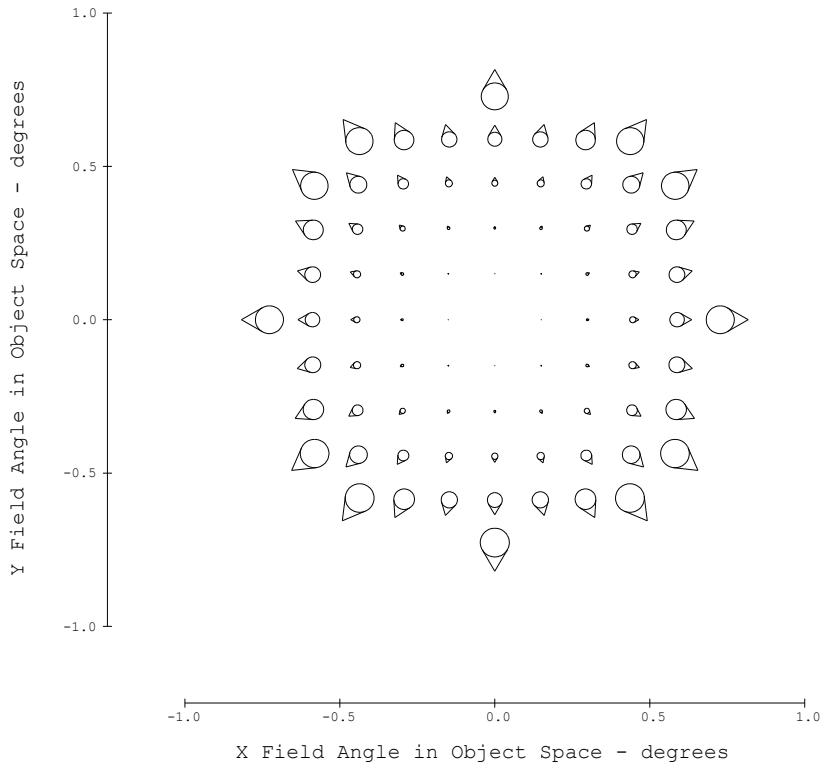




Afocal TMA

JCP 22-Sep-16





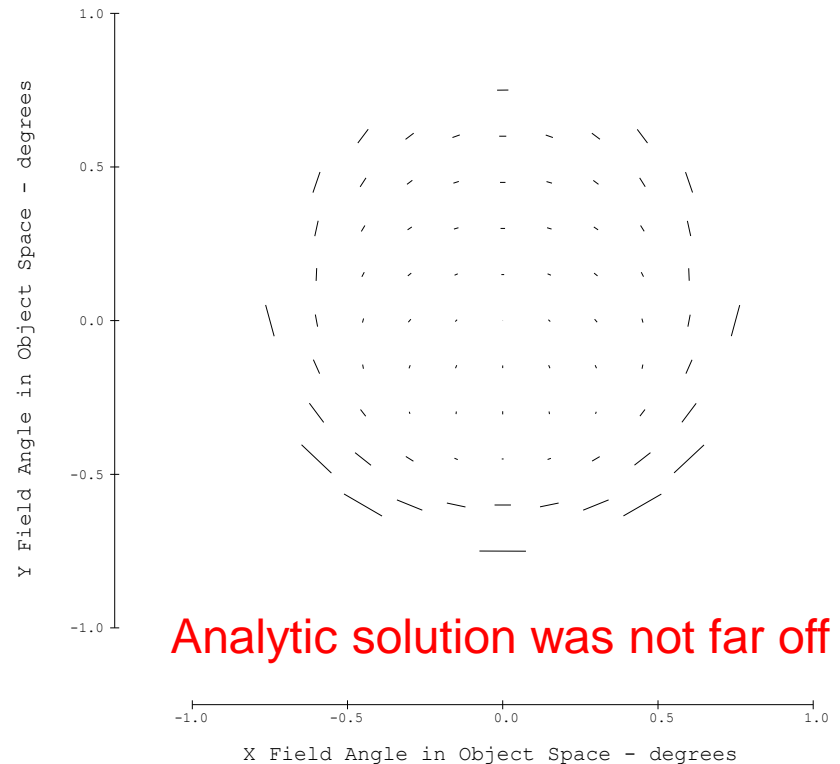
FRINGE ZERNIKE PAIR 27 AND 28  
vs  
FIELD ANGLE IN OBJECT SPACE

Minimum = 0.57266e-8  
Maximum = 0.005145  
Average = 0.0021241  
Std Dev = 0.0016179

Afocal TMA

JCP 22-Sep-16

With 0.015° (~1 arcmin) more tilt



FRINGE ZERNIKE PAIR 25 AND 26  
vs  
FIELD ANGLE IN OBJECT SPACE

Minimum = 0.26156e-7  
Maximum = 0.028889  
Average = 0.007245  
Std Dev = 0.006869

Afocal TMA

0.079waves ( 587.6 nm)

Analytic solution was not far off  
Dropped from Max of ~0.2 waves  
to ~0.03 waves

Worst field point has RMSWE 0.018λ

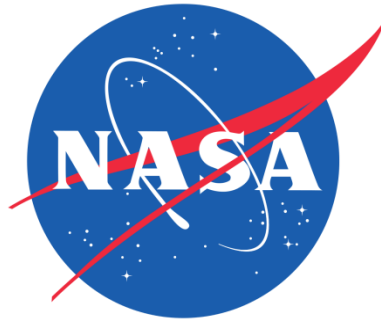


# Conclusion

- Several analytical starting point design methods are being developed to facilitate a survey of the four-mirror freeform solution space.
- A combination of these methods can allow for unobscured starting points that are corrected for third order image degrading aberrations.



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**Questions?**

