Estimation of SOC of $LiFePO_4$ Cells Using a Reduced Order Model with Extended Kalman Filter

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Outline

Background

- SOC estimation and application
- Need for a reduced order model

Modeling of LFP cells and order reduction

- Modeling of LFP cells
- Order reduction
- Validation of the ROM

Design of EKF

- Principle of EKF
- Results of the ROM with EKF

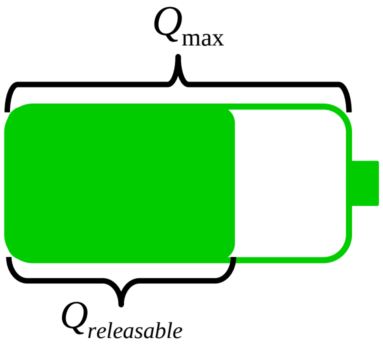
Summary

SOC estimation and application

Definition of SOC

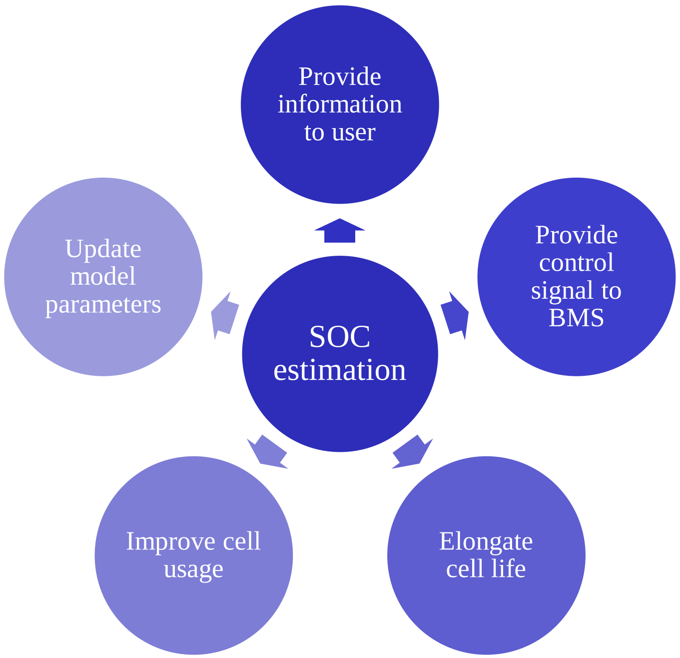
The ratio of releasable charges stored in a battery ($Q_{releasable}$) and the maximum capacity of the battery (Q_{max})

$$SOC = \frac{Q_{releasable}}{Q_{\max}}$$



Application of SOC estimation

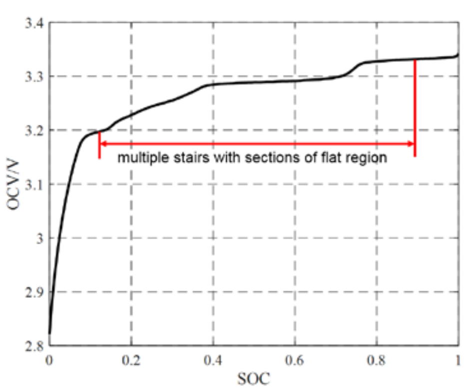
- Provide information to user
  - Available energy
  - Recharge
- Provide information to BMS
  - Energy management
  - Detect cell balancing
- Elongate cell life
  - Prevent overcharge
  - Prevent overdischarge
- Improve cell usage
  - Usage indication
- Update model parameters



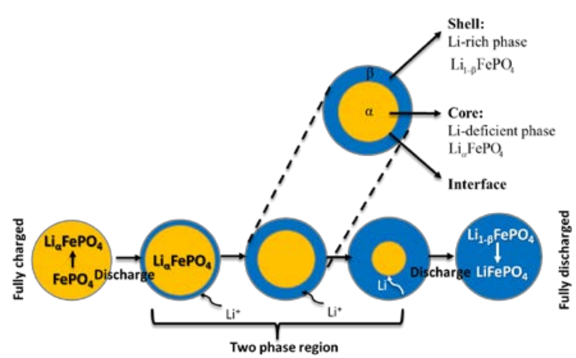
Need of a reduced order model

Estimation method	Open loop SOC estimation	Closed loop SOC estimation with ECM	Closed loop SOC estimation with ROM
Pros	• Minimum requirement of CPU	• Simple model structure • Can track SOC even if the initial condition is unknown	✓ Accurately model the voltage plateau and path dependency ✓ Can track SOC even if the initial condition is unknown
Cons	• Inaccurate estimation if initial SOC is unknown	• Circuit components cannot reflect the physical states • Inaccurate modeling for **voltage plateau, phase transition** and **path dependency**	• Complex modeling structure

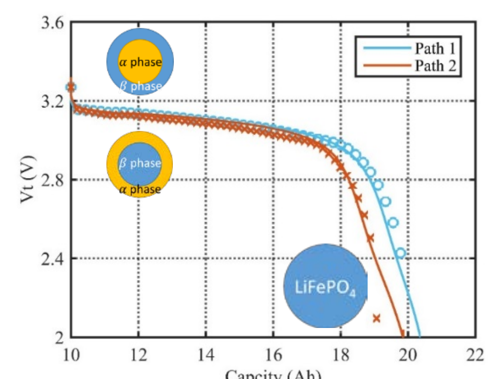
OCV characteristic of a LFP cell



Two phase transition



Path dependency



Modeling of LFP cells – Two phase transition

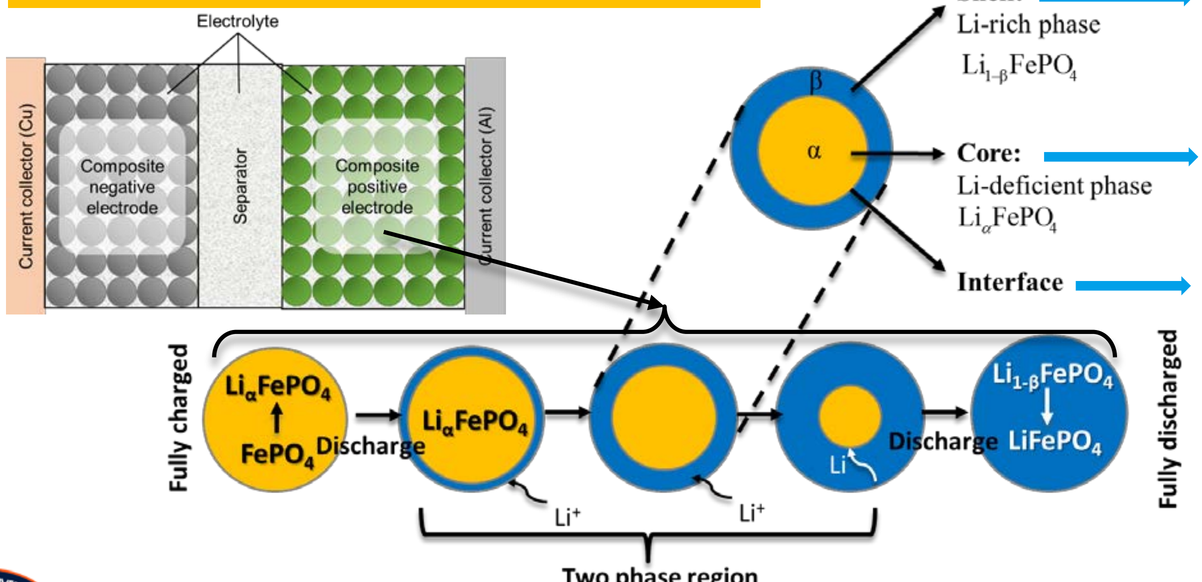
Modeling principle

Input: I, V, T_{amb}, SOC_0

Internal variables		Physical Principle
Ion concentration in electrode (C_s)	→	• Mass conservation
Ion concentration in electrolyte (C_e)	→	• Ohm's law
Potential in electrode (ϕ_s)	→	• Charge conservation
Potential in electrolyte (ϕ_e)	→	• Electrochemical kinetics
Heat generation	→	• Energy equation

Output: SOC, V_t, $T_{battery}$

Modeling of two phase transition



Fick's law of diffusion: $\frac{\partial C_s}{\partial t} = \frac{D_{s,\beta}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_s}{\partial r}\right)$

Boundary condition: $D_{s,\beta}\frac{\partial C_s}{\partial r}\Big|_{r=R_s} = \frac{-j^{Li}}{a_s F}$

Fick's law of diffusion: $\frac{\partial C_s}{\partial t} = \frac{D_{s,\alpha}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_s}{\partial r}\right)$

Boundary condition: $D_{s,\alpha}\frac{\partial C_s}{\partial r}\Big|_{r=0} = 0$

Mass conservation in the control volume:

Generation $= -(C_\beta - C_\alpha)\mathrm{d}r_0$

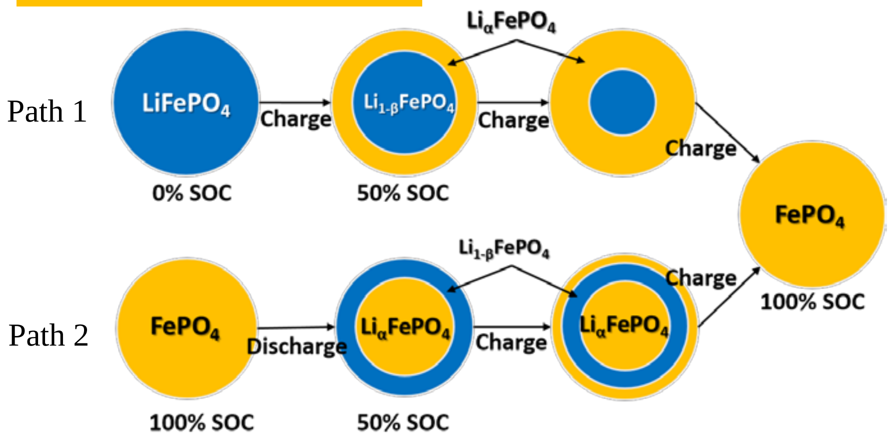
Flowin – Flowout =

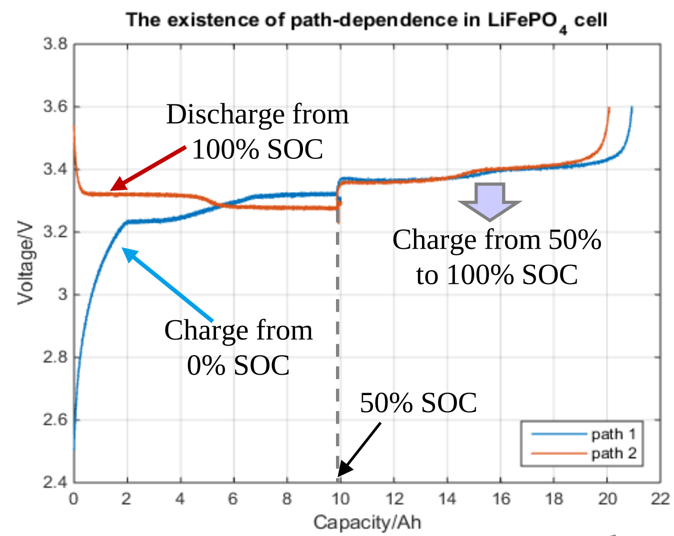
$$\left(D_{s,\beta}\frac{\partial C_{s,\beta}}{\partial r}\Big|_{r=r_0} - D_{s,\alpha}\frac{\partial C_{s,\alpha}}{\partial r}\Big|_{r=r_0}\right)\mathrm{d}t$$

$$-(C_\beta - C_\alpha)\frac{\mathrm{d}r_0}{\mathrm{d}t} = D_{s,\beta}\frac{\partial C_{s,\beta}}{\partial r}\Big|_{r=r_0} - D_{s,\alpha}\frac{\partial C_{s,\alpha}}{\partial r}\Big|_{r=r_0}$$

Modeling of LFP cells - Path dependency

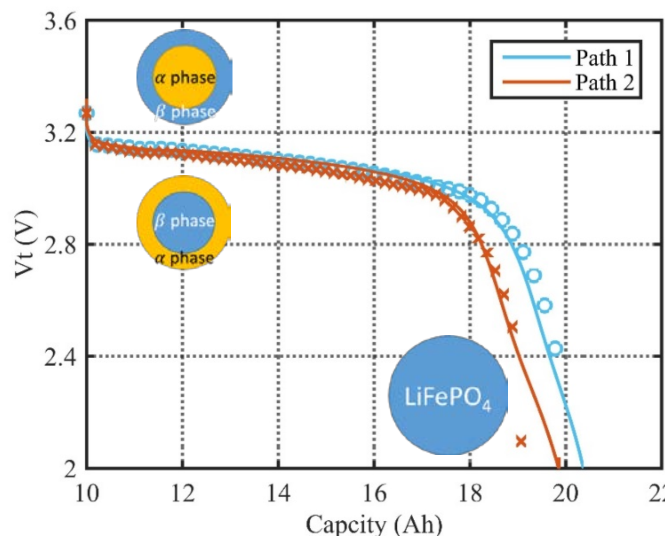
Path dependency





Validation of path dependency

- ❖ Testing condition
- Path1
  - Previously discharged with 0.1C from 100% to 50% SOC
- Path 2
  - Previously charged with 0.1C from 0% to 50% SOC



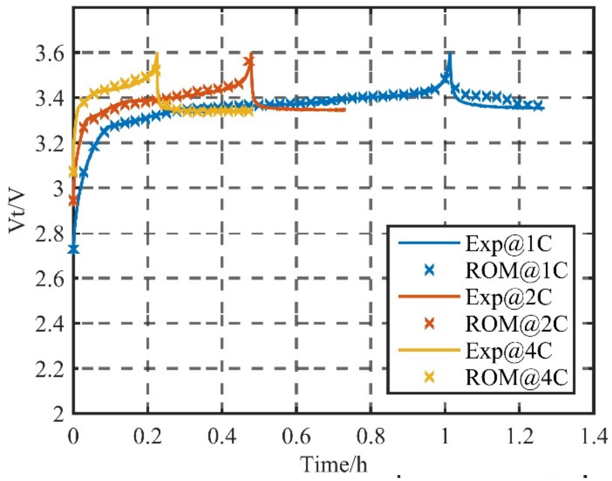
- ❖ Different path results in different releasable capacity.

Order reduction

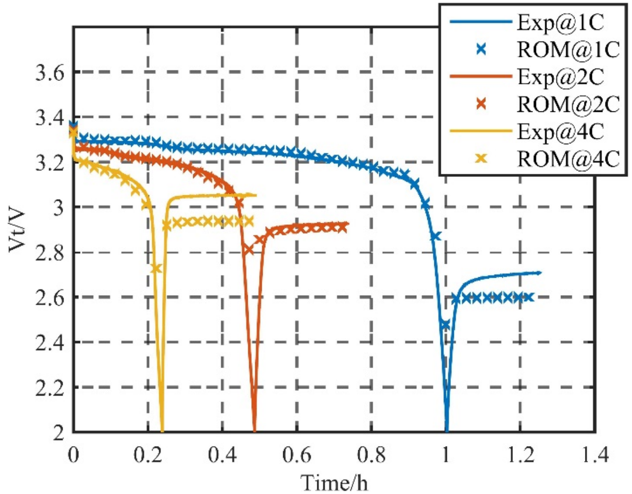
	Full order model (FOM)	Reduction technique	Reduced order model (ROM)
Ion concentration in electrode	$\frac{\partial c_s}{\partial t} = \frac{D_{s,\beta}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial c_s}{\partial r}\right); D_{s,\beta}\frac{\partial c_s}{\partial r}\Big\|_{r=r_1} = 0$ $\frac{\partial c_s}{\partial t} = \frac{D_{s,\alpha}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial c_s}{\partial r}\right); D_{s,\alpha}\frac{\partial c_s}{\partial r}\Big\|_{r=R_s} = \frac{-j^{Li}}{a_s F}$ $\left(c_{s,\alpha\beta} - c_{s,\beta\alpha}\right)\frac{\mathbf{d}r_0}{\mathbf{d}t} = D_{s,\beta}\frac{\partial c_{s,\beta}}{\partial r}\Big\|_{r=r_0} - D_{s,\alpha}\frac{\partial c_{s,\alpha}}{\partial r}\Big\|_{r=r_0}$	**Polynomial approach**	$\frac{d}{dt}c_{s,ave} - 3\frac{D_s}{r_0^2}\left(35(c_{s,surf} - c_{s,ave}) - 8q_{ave}r_0\right) = 0$ $(c_{s,\beta\alpha} - c_{s,\alpha\beta})\frac{dr_0}{dt} = -D_{s,\beta}\frac{(c_{s,surf} - c_{s,\beta\alpha})k_2 - (c_{s,surf} - c_{s,\beta\alpha})k_4}{k_2 k_3 - k_1 k_4}$ $+\frac{D_s}{r_0}\left(35(c_{s,surf} - c_{s,ave}) - 8q_{ave}r_0\right)$
Ion concentration in electrolyte	$\frac{\partial(\varepsilon_e c_e)}{\partial t} = \frac{\partial}{\partial x}\left(D_e^{eff}\frac{\partial}{\partial x}c_e\right) + \frac{1-t_+^0}{F}j^{Li}$ $\frac{\partial c_e}{\partial t}\Big\|_{x=0} = \frac{\partial c_e}{\partial t}\Big\|_{x=L} = 0$	**Residual grouping**	$\dot{c}_e = \hat{A}\cdot c_e + \hat{B}\cdot I$ $y = \hat{C}\cdot c_e + \hat{D}\cdot I$
Ohm's law in electrolyte	$\frac{\partial}{\partial x}\left(\kappa^{eff}\frac{\partial}{\partial x}\phi_e\right) + \frac{\partial}{\partial x}\left(\kappa_D^{eff}\frac{\partial}{\partial x}\ln c_e\right) + j^{Li} = 0$ $\frac{\partial \phi_e}{\partial x}\Big\|_{x=0} = \frac{\partial \phi_e}{\partial x}\Big\|_{x=L} = 0$	**C_e has no influence on reaction current** $\frac{\partial}{\partial x}\left(\kappa_D^{eff}\frac{\partial \ln c_e}{\partial x}\right) = 0$	$\frac{\partial}{\partial x}\left(\kappa^{eff}\frac{\partial \phi_e}{\partial x}\right) + j^{Li} = 0$
Electrochemical kinetics	$j^{Li} = a_s i_0\left\{\exp\left[\frac{\alpha_a F}{RT}\eta\right] - \exp\left[-\frac{\alpha_c F}{RT}\eta\right]\right\}$ $\eta = \phi_s - \phi_e - U$	**Linearization**	$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\phi_{se}\right) = j^{Li}\left(\frac{1}{\sigma^{eff}} + \frac{1}{\kappa^{eff}}\right)$ $j^{Li} = \frac{a_s i_0 F}{RT}(\phi_{se} - U)$

Validation of ROM

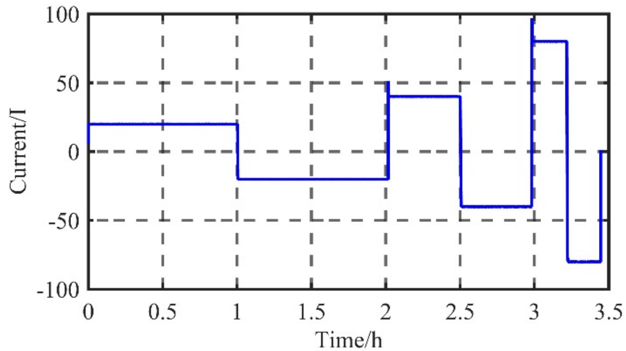
Test condition:
Profile: CC charge
Temperature: 25°C
Current: 1C,2C, 4C
Initial SOC: 0%

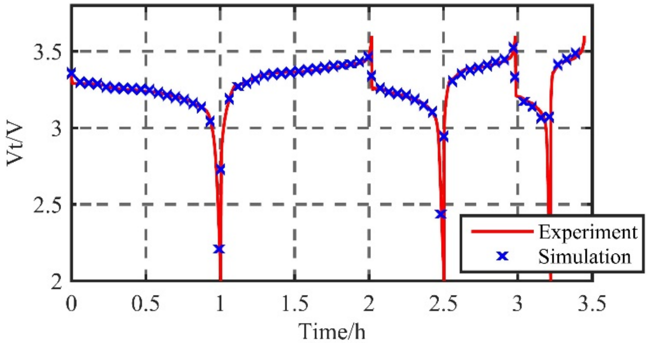


Test condition:
Profile: CC discharge
Temperature: 25°C
Current: 1C,2C, 4C
Initial SOC: 100%



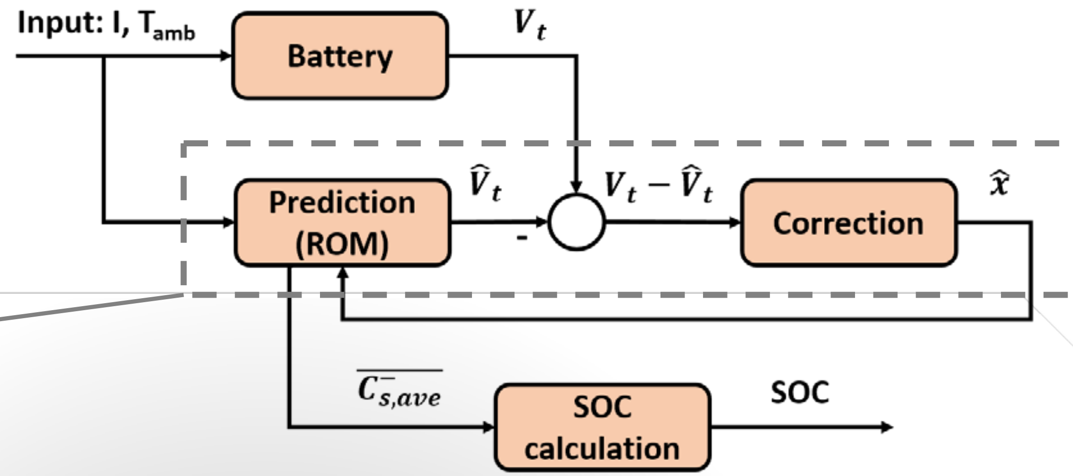
Test condition:
Profile: Multiple cycle
Temperature: 25°C
Current: 1C,2C, 4C
Initial SOC: 100%





Principle of EKF

Working principle of EKF Based on ROM:



EKF Algorithm

Battery Model

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1})$$

$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{v}_k)$$

Initialization

State Estimation:

$\hat{x}_0$

Error Covariance:

P_0

Time Update (Prediction)

State:

$$\hat{x}_k^- = f(\hat{x}_k^-, u_{k-1}, 0)$$

Error Covariance:

$$P_k^- = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T$$

Measurement Update (Correction)

Kalman Gain:

$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + V_k P_{k-1} V_k^T\right)^{-1}$$

State:

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$$

Error Covariance

$$P_k = (I - K_k H_k) P_k^-$$

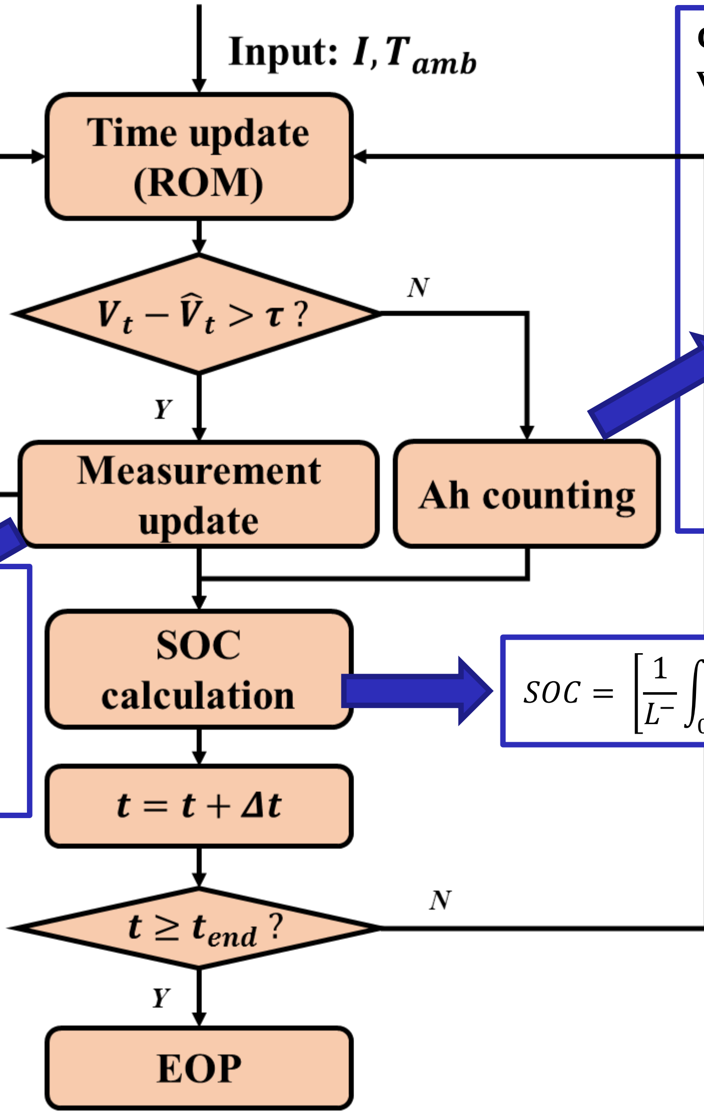
Flowchart of SOC Estimation Process

State: $x = [\overline{c_{s,ave}^-}]$
Measurement: $y = [V_t]$

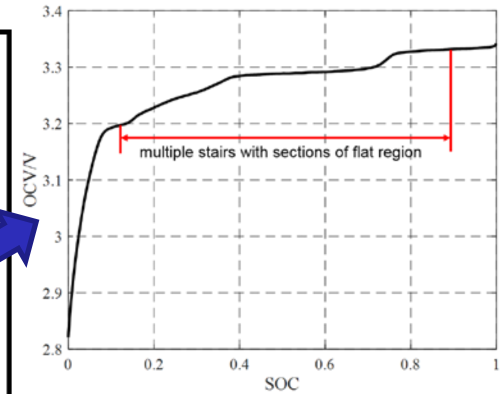
Model Reformulation:

$$\overline{c_{s,ave}^-}^{k} = \overline{c_{s,ave}^-}^{k-1} - 3\frac{j^{Li}}{R_s a_s F}\frac{I}{AL^-}$$

$$V_t = U^+(\bar{y}) - U^-(\bar{x}) - \eta$$



Combined method to deal with voltage plateau



Jacobians:

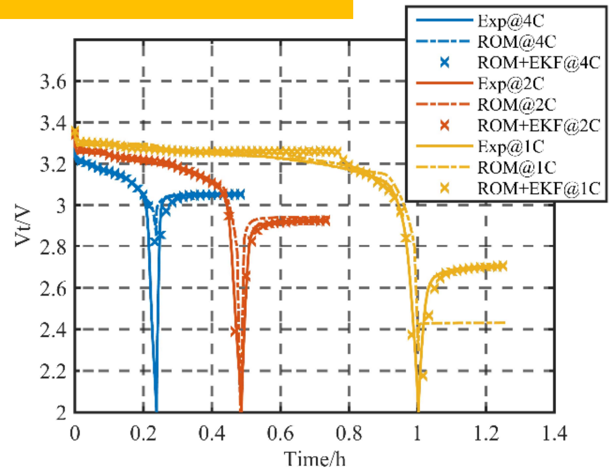
$$A = \frac{\partial \overline{c_{s,ave}^-}^{k}}{\partial \overline{c_{s,ave}^-}^{k-1}} = \mathrm{I}$$

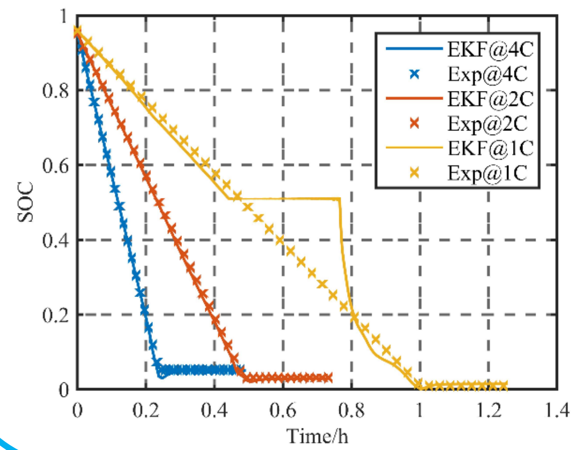
$$H = \frac{\partial V_t}{\partial \overline{c_{s,ave}^-}} = \frac{\partial U_{OCV}^+}{\partial \bar{y}}\frac{\partial \bar{y}}{\partial \overline{c_{s,ave}^+}}\frac{\partial \overline{c_{s,ave}^+}}{\partial \overline{c_{s,ave}^-}} - \frac{\partial U_{OCV}^-}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial \overline{c_{s,ave}^-}}$$

$$SOC = \left[\frac{1}{L^-}\int_0^{L^-}\frac{c_{s,ave\_} - c_{s,max}\cdot x_0}{c_{s,max}(x_{100} - x_0)}dx\right]\cdot 100\%$$

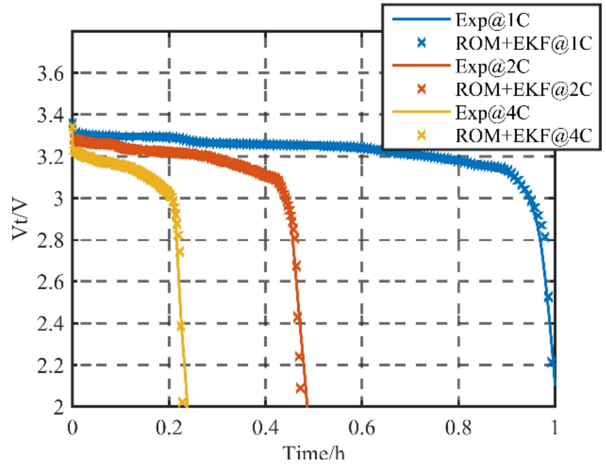
Result of ROM with EKF

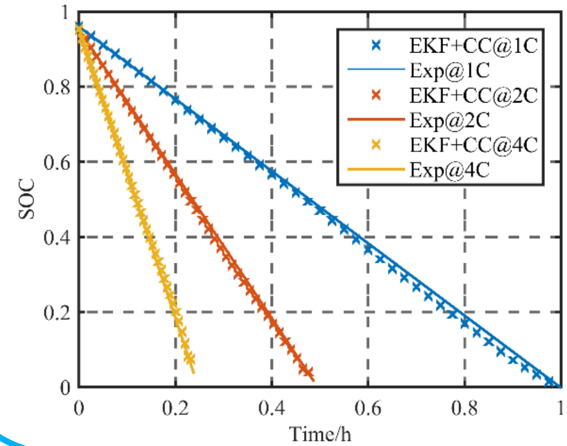
ROM with EKF



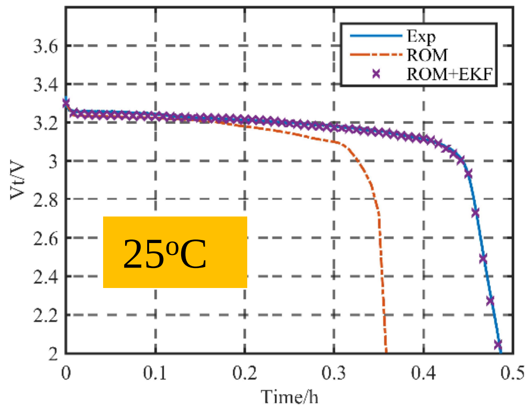


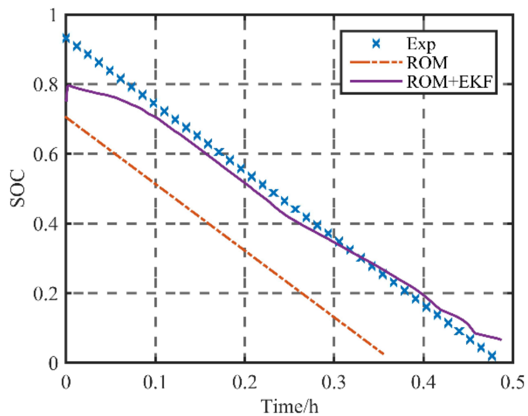
ROM with Combined method





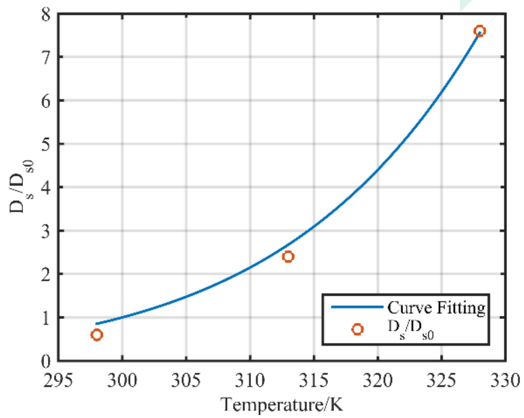
Validation of ROM + EKF (with initial error and temperature dependency)



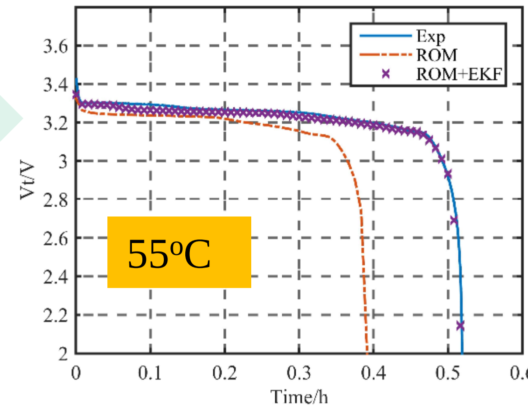


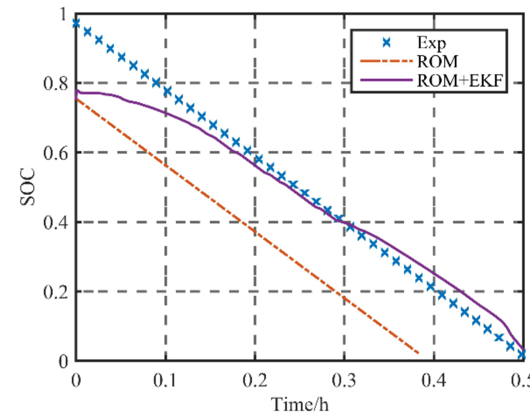
Arrhenius equation

$$D_s = D_{s0}\cdot \exp\left(\frac{E_a}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)\right)$$



Curve fitting of different temperature





- ❖ Test condition: 2C-rate CC discharge at 25ºC.
- ❖ 20% initial SOC error (0.2V terminal voltage error) is assumed.
- ❖ The initial error can be corrected with EKF.

Summery

❑ Summary

- Development of ROM for $LiFePO_4$ cells considering **two phase transition** and **path dependency**.
- Development of **combined EKF algorithm** for SOC estimation.
- Comparison of simulation results of combined algorithm and tradition EKF and their analysis under different operation conditions.

❑ Conclusion

- The combined algorithm shows a better SOC estimation result.

❑ Future work

- Incorporation of **thermal model**.
- Validation of **drive cycle** profiles

Thanks for your attention!