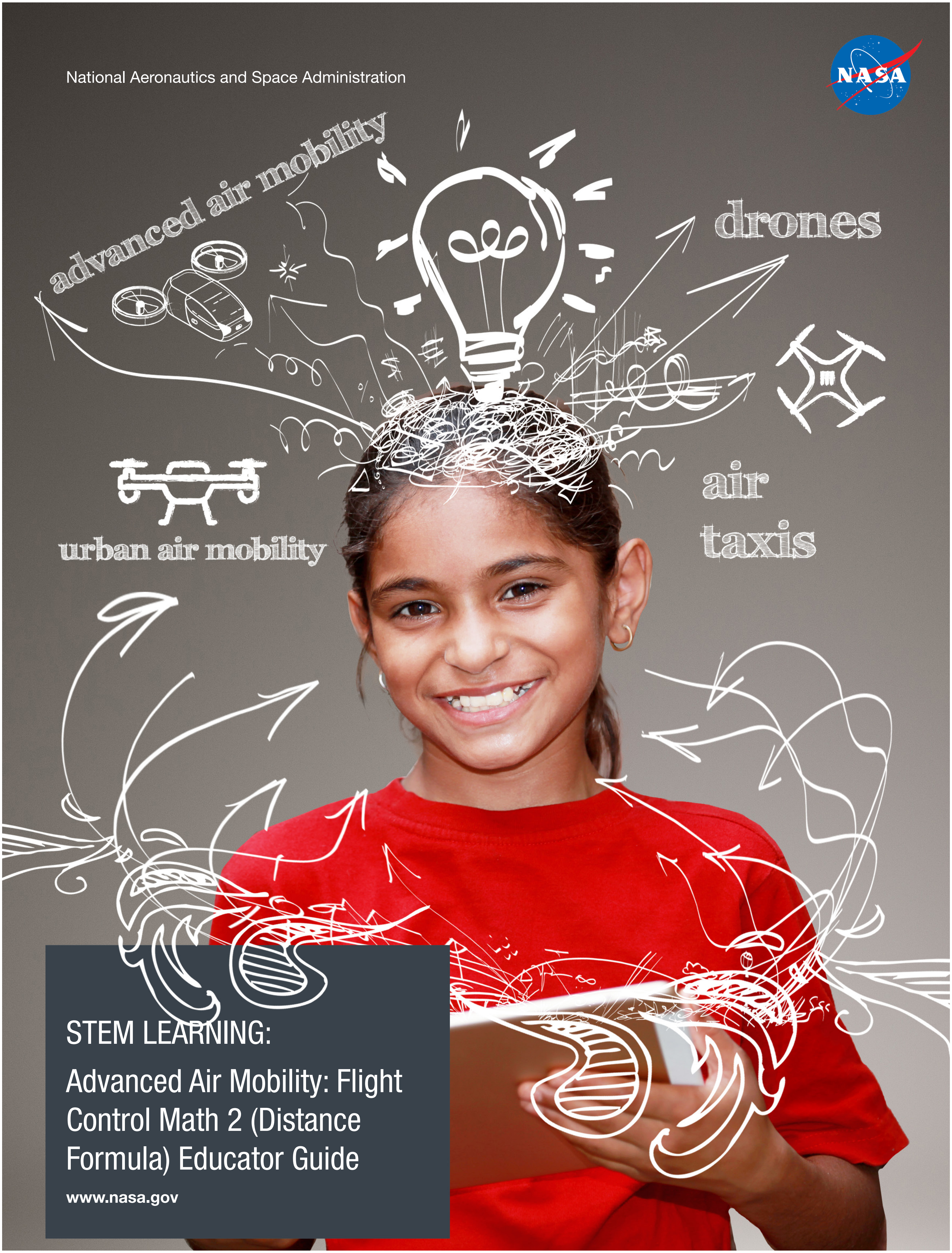
National Aeronautics and Space Administration



STEM LEARNING:

Advanced Air Mobility: Flight Control Math 2 (Distance Formula) Educator Guide

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OVERVIEW

In this lesson, students are introduced to the role of package delivery drones in Advanced Air Mobility, or AAM. Using the distance formula, students calculate the distance between given coordinates and use their results to determine the feasibility of utilizing package delivery drones in different scenarios.

Objectives

Students will be able to:

- Use the distance formula to calculate the distance between two points
- Evaluate the effectiveness of using package delivery drones in different scenarios based on data

Standards

CCS.MATH.CONTENT.8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Next Generation Science Standards

Science and Engineering Practices

- Engaging in argument from evidence
- Analyzing and interpreting data

Student Prerequisite Knowledge

Before beginning this lesson, students should be familiar with:

- Obtaining coordinates of points plotted on a graph
- Plotting given coordinates on a graph
- Using the distance formula to determine the distance between two points

Materials

- Student Guide (one per student)

Lesson Modifications

Two options are available for this lesson: one has all the points in quadrant 1 of the coordinate plane and the other has them in all four quadrants. This guide provides answers for both. Print out the student guide(s) for whichever version is best suited for the classes working on it.

This lesson can easily be further modified depending on student needs. More locations can be added to the graph to provide more practice. Locations can be chosen to make calculations more challenging. Some ideas include:

- Less challenging: add a delivery location that has the same (x) or (y) coordinate as the Package Delivery Center that will be delivering. Mathematically, this zeroes out one portion of the distance formula.
- Less challenging: add a location with both the (x) and (y) coordinates higher than the (x) and (y) coordinates of the Package Delivery Center that will be delivering. This eliminates negative numbers from occurring when using the distance formula.
- More challenging: add locations with coordinates requiring decimals. One example is a delivery location at the coordinates (35.5, 18.5).

The difficulty can be varied by allowing or prohibiting the use of calculators. If calculators are not used, answers will need to be expressed as square roots unless locations are added that lead to perfect squares. One such location would be at the coordinates (40, 60). When the distance formula is used for this point and Package Delivery Center 1, the answer is exactly 25 squares.

If students are familiar with the Pythagorean Theorem, this lesson provides a good opportunity to point out to them that the distance formula is the same thing.

Grouping Students

This activity is designed to be completed independently. Grouping can, however, be used if necessary. Students working in pairs can independently solve the problems and then compare answers to validate their solutions or find errors.

Steps

1. Use a warm-up or other method to introduce the students to the concept of AAM and the use of package delivery drones. A video showing package delivery drones can also be shown.
2. Review the distance formula with students to ensure they understand how to use it.
3. Show the map of Dallas, Texas without the grid and locations and discuss some of the issues of delivering packages in a dense, urban setting like this.
4. Show the map of Dallas, Texas with the locations marked but without the grid. Discuss how the distance formula can be used in this scenario.
5. Show the map with the grid and locations on it. Discuss how the distance formula can now be used. Also, review the scale and emphasize that when calculating distances, it is important to convert answers from squares to kilometers.
6. Distribute the student guide(s) and allow students to begin working. Students can work with the graph that shows the map or the one without the map.
7. Allow students to verify answers for the “Getting the Coordinates” section. (This allows students to find errors that would be compounded in the “Using the Distance Formula” section.)

ANSWERS (QUADRANT 1)

PART 1: GETTING THE COORDINATES

To determine the distances between points on a map, a graph has been placed over the map. In figure 2, the two package distribution centers and five customers' houses have been plotted on the graph.

The origin (0, 0) is the company's headquarters and is indicated on the graph.

Determine the coordinates of each location and enter them below:

Location	Coordinate (x)	Coordinate (y)
Package Distribution Center 1	9	19
Package Distribution Center 2	24	2
Anthony	8	2
Barbara	14	15
Clive	20	12
Deirdre	21	18
Ekani	30	9

PART 2: USING THE DISTANCE FORMULA

Your delivery drones cannot fly too far because of their batteries. So, it is very important that you schedule deliveries to come from the distribution center closest to the final destination. Your system determined that Barbara and Deirdre are closest to Package Distribution Center 1, while Anthony and Ekani are closest to Package Distribution Center 2.

Note: Don't forget to convert your answers into kilometers (km) (1 square = 0.5 km).

1. What is the distance from Package Distribution Center 1 to Barbara's house?

$$distance = \sqrt{(14-9)^2 + (15-19)^2} = \sqrt{25+16} = \sqrt{41} = 6.4 \text{ squares}$$

$$distance = (6.4)(0.5) = 3.2 \text{ km}$$

2. What is the distance from Package Distribution Center 1 to Deirdre's house?

$$distance = \sqrt{(21-9)^2 + (18-19)^2} = \sqrt{144+1} = \sqrt{145} = 12 \text{ squares}$$

$$distance = (12)(0.5) = 6.0 \text{ km}$$

3. What is the distance from Package Distribution Center 2 to Anthony's house?

$$distance = \sqrt{(8-24)^2 + (2-2)^2} = \sqrt{256+0} = \sqrt{256} = 16 \text{ squares}$$

$$distance = (16)(0.5) = 8.0 \text{ km}$$

4. What is the distance from Package Distribution Center 2 to Ekani's house?

$$distance = \sqrt{(30-24)^2 + (9-2)^2} = \sqrt{36+49} = \sqrt{85} = 9.2 \text{ squares}$$

$$distance = (9.2)(0.5) = 4.6 \text{ km}$$

5. Your boss calls and asks which distribution center is closest to Clive's house. You need to figure out the answer and call her back. Make sure you have mathematical calculations to use as evidence to support your claim.

Clive is closer to Package Delivery Center 2
Calculate the distance from Package Delivery Center 1:

$$distance = \sqrt{(20-9)^2 + (12-19)^2} = \sqrt{121+49} = \sqrt{170} = 13 \text{ squares}$$

$$distance = (13)(0.5) = 6.5 \text{ km}$$

Calculate the distance from Package Delivery Center 2:

$$distance = \sqrt{(20-24)^2 + (12-2)^2} = \sqrt{16+100} = \sqrt{116} = 10.8 \text{ squares}$$

$$distance = (10.8)(0.5) = 5.4 \text{ km}$$

6. A new customer calls to see if it's possible to have your drone delivered to his house. The coordinates of his house are (35, 21). Plot this on the graph.

   a. If your drone is capable of traveling 25 km (round trip), would it be capable of delivering to this location from Package Delivery Center 1? (Keep in mind that the drone must fly back to the delivery center as well.) Support your answer with your calculations.

$$distance = \sqrt{(35-9)^2 + (21-19)^2} = \sqrt{676+4} = \sqrt{680} = 26.1 \text{ squares}$$

$$distance = (26.1)(0.5) = 13.1 \text{ km}$$

To calculate round trip distance: round trip distance = 13.1 × 2 = 26.2 km
26.2 > 25, so it is not capable.

b. If your drone is capable of traveling 25 km (round trip), would it be capable of delivering to this location from Package Delivery Center 1? (Keep in mind that the drone must fly back to the delivery center as well.) Support your answer with your calculations.

$$distance = \sqrt{(35-24)^2 + (21-2)^2} = \sqrt{121+361} = \sqrt{482} = 22 \text{ squares}$$

$$distance = (22)(0.5) = 11.0 \text{ km}$$

To calculate round trip distance: round trip distance = 11 × 2 = 22.0 km

22 < 25, so it is capable.

7. Keeping in mind that the maximum range of the drone is 25 km, would it be possible to leave Package Delivery Center 1, fly to Barbara's house, then fly to Deirdre's house, and finally return to Package Delivery Center 1? Support your answer with your calculations.

Distance from Package Delivery Center 1 to Barbara's house: 3.2 km (from question 1)

Distance from Deirdre's house to Package Delivery Center 1: 6.0 km (from question 2)

Distance from Barbara's house (14, 15) to Deirdre's house (21, 18):

$$distance = \sqrt{(21-14)^2 + (18-15)^2} = \sqrt{49+9} = \sqrt{58} = 7.6 \text{ squares}$$

$distance = (7.6)(0.5) = 3.8$ km

$total\ distance = 3.2 + 3.8 + 6.0 = 13.0$ km

13 < 25 , so it is possible.

ANSWERS (ALL QUADRANTS)

PART 1: GETTING THE COORDINATES

To determine the distances between points on a map, a graph has been placed over the map. In figure 2, the two package distribution centers and five customers' houses have been plotted on the graph.

The origin (0, 0) is the company's headquarters and is indicated on the graph.

Determine the coordinates of each location and enter them below:

Location	Coordinate (x)	Coordinate (y)
Package Distribution Center 1	-6	10
Package Distribution Center 2	9	-7
Anthony	-7	-7
Barbara	-1	6
Clive	5	3
Deirdre	6	9
Ekani	15	0

PART 2: USING THE DISTANCE FORMULA

Your delivery drones cannot fly too far because of their batteries. So, it is very important that you schedule deliveries to come from the distribution center closest to the final destination. Your system determined that Barbara and Deirdre are closest to Package Distribution Center 1, while Anthony and Ekani are closest to Package Distribution Center 2.

Note: Don't forget to convert your answers into kilometers (km) (1 square = 0.5 km).

1. What is the distance from Package Distribution Center 1 to Barbara's house?

$$distance = \sqrt{((-1)-(-6))^2 + (6-10)^2} = \sqrt{25+16} = \sqrt{41} = 6.4 \text{ squares}$$

$$distance = (6.4)(0.5) = 3.2 \text{ km}$$

2. What is the distance from Package Distribution Center 1 to Deirdre's house?

$$distance = \sqrt{(6-(-6))^2 + (9-10)^2} = \sqrt{144+1} = \sqrt{145} = 12 \text{ squares}$$

$$distance = (12)(0.5) = 6.0 \text{ km}$$

3. What is the distance from Package Distribution Center 2 to Anthony's house?

$$distance = \sqrt{((-7) - 9)^2 + ((-7) - (-7))^2} = \sqrt{256 + 0} = \sqrt{256} = 16 \text{ squares}$$

$$distance = (16)(0.5) = \mathbf{8.0 \text{ km}}$$

4. What is the distance from Package Distribution Center 2 to Ekani's house?

$$distance = \sqrt{(15 - 9)^2 + (0 - (-7))^2} = \sqrt{36 + 49} = \sqrt{85} = 9.2 \text{ squares}$$

$$distance = (9.2)(0.5) = \mathbf{4.6 \text{ km}}$$

5. Your boss calls and asks which distribution center is closest to Clive's house. You need to figure out the answer and call her back. Make sure you have mathematical calculations to use as evidence to support your claim.

Clive is closer to Package Delivery Center 2
Calculate the distance from Package Delivery Center 1:

$$distance = \sqrt{(5 - (-6))^2 + (3 - 10)^2} = \sqrt{121 + 49} = \sqrt{170} = 13 \text{ squares}$$

$$distance = (13)(0.5) = \mathbf{6.5 \text{ km}}$$

Calculate the distance from Package Delivery Center 2:

$$distance = \sqrt{(5 - 9)^2 + (3 - (-7))^2} = \sqrt{16 + 100} = \sqrt{116} = 10.8 \text{ squares}$$

$$distance = (10.8)(0.5) = \mathbf{5.4 \text{ km}}$$

6. A new customer calls to see if it's possible to have your drone deliver to his house. The coordinates of his house are (20, 12). Plot this on the graph.

   a. If your drone is capable of traveling 25 km (round trip), would it be capable of delivering to this location from Package Delivery Center 1? (Keep in mind that the drone must fly back to the delivery center as well.) Support your answer with your calculations.

$$distance = \sqrt{(20 - (-6))^2 + (12 - 10)^2} = \sqrt{676 + 4} = \sqrt{680} = 26.1 \text{ squares}$$

$$distance = (26.1)(0.5) = \mathbf{13.1 \text{ km}}$$

To calculate round trip distance: *round trip distance* = 13.1 × 2 = 26.2 km
26.2 > 25, so it is not capable.

b. If your drone is capable of traveling 25 km (round trip), would it be capable of delivering to this location from Package Delivery Center 2? (Keep in mind that the drone must fly back to the delivery center as well.) Support your answer with your calculations.

$$distance = \sqrt{(20-9)^2 + (12-(-7))^2} = \sqrt{121+361} = \sqrt{482} = 22 \text{ squares}$$

$$distance = (22)(0.5) = 11.0 \text{ km}$$

To calculate round trip distance: round trip distance = 11 × 2 = 22.0 km

22 < 25, so it is capable.

7. Keeping in mind that the maximum range of the drone is 25 km, would it be possible to leave Package Delivery Center 1, fly to Barbara's house, then fly to Deirdre's house, and finally return to Package Delivery Center 1? Support your answer with your calculations.

Distance from Package Delivery Center 1 to Barbara's house: 3.2 km (from question 1)

Distance from Deirdre's house to Package Delivery Center 1: 6.0 km (from question 2)

Distance from Barbara's house (-1, 6) to Deirdre's house (6, 9):

$$distance = \sqrt{(6-(-1))^2 + (9-6)^2} = \sqrt{49+9} = \sqrt{58} = 7.6 \text{ squares}$$

$distance = (7.6)(0.5) = 3.8$ km

$total\ distance = 3.2 + 3.8 + 6.0 = 13.0$ km

13 < 25 , so it is possible.

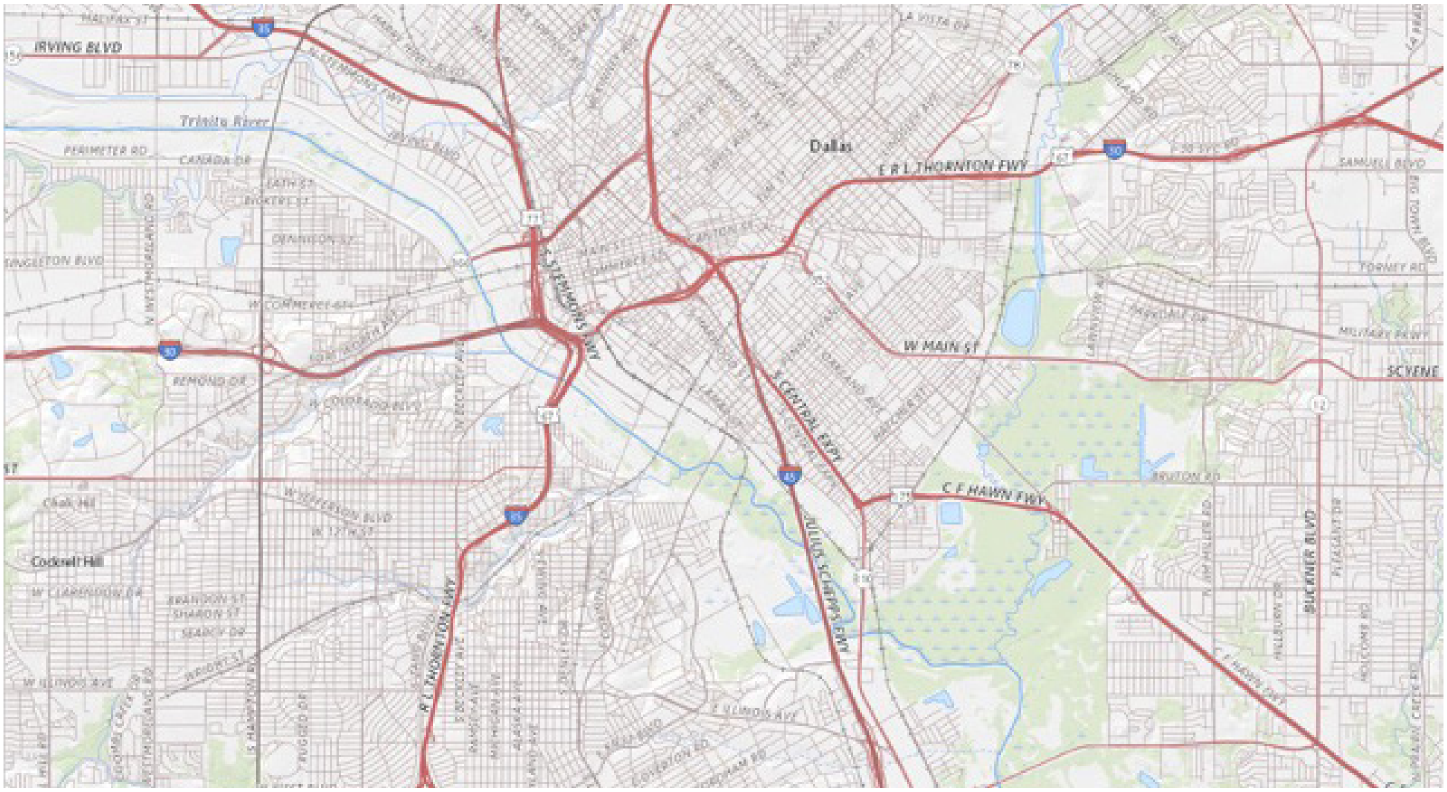

Figure 1. Map of Dallas, Texas. Credit: USGS

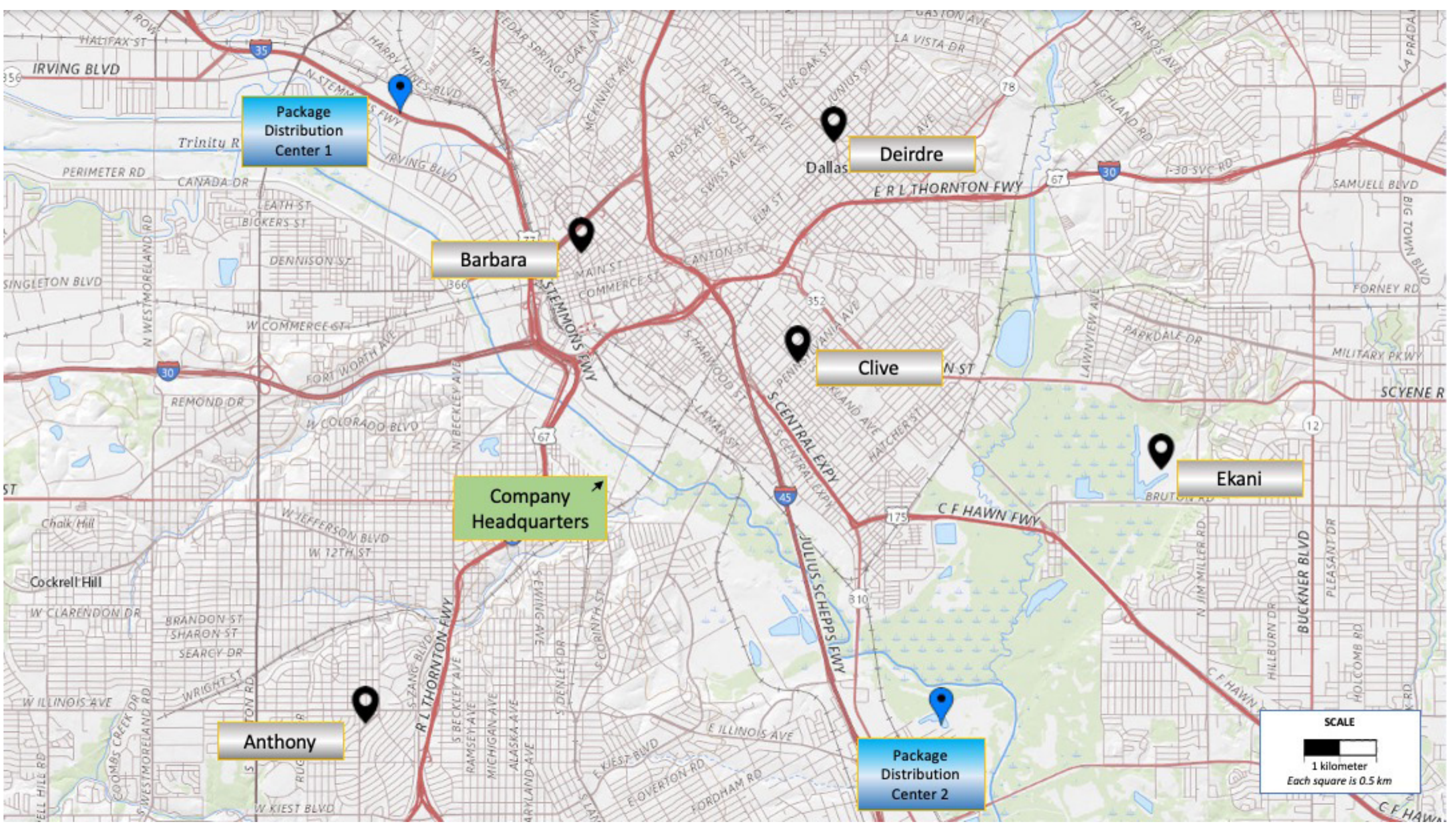

Figure 2. Map of Dallas, Texas with the locations marked. Credit: USGS

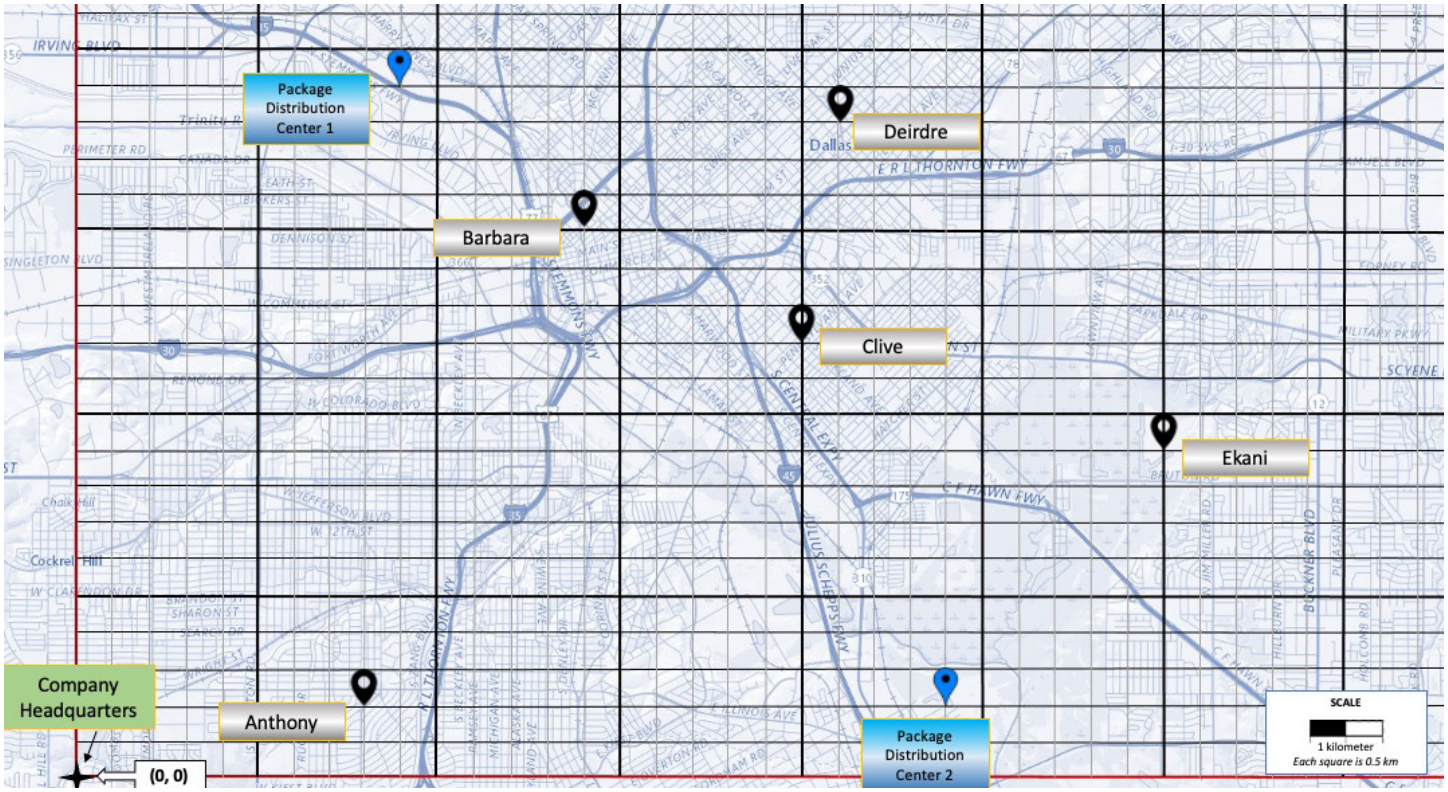

Figure 3. Grid with map for quadrant 1 only. Credit: USGS

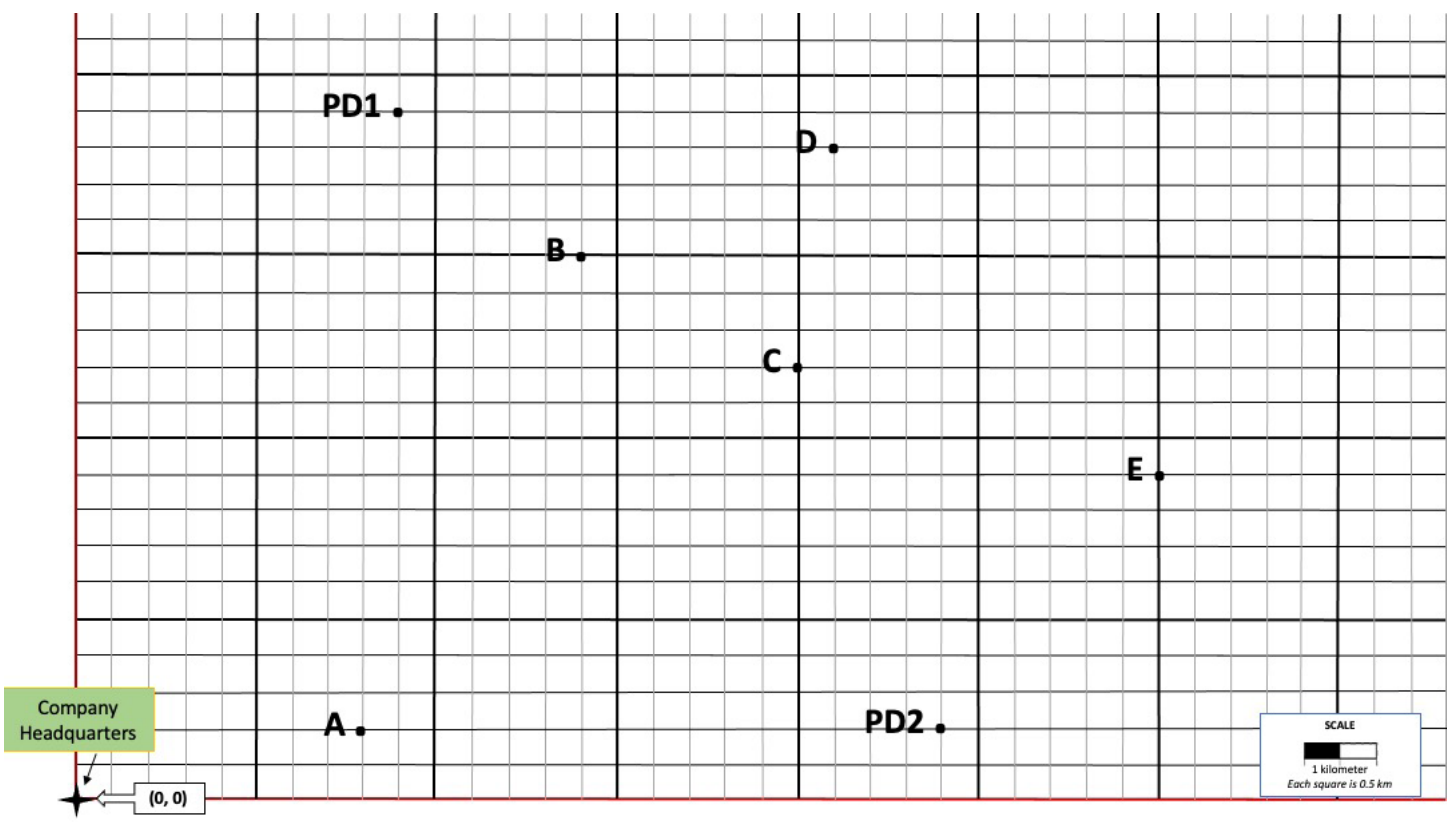

Figure 4. Grid without map for quadrant 1 only.

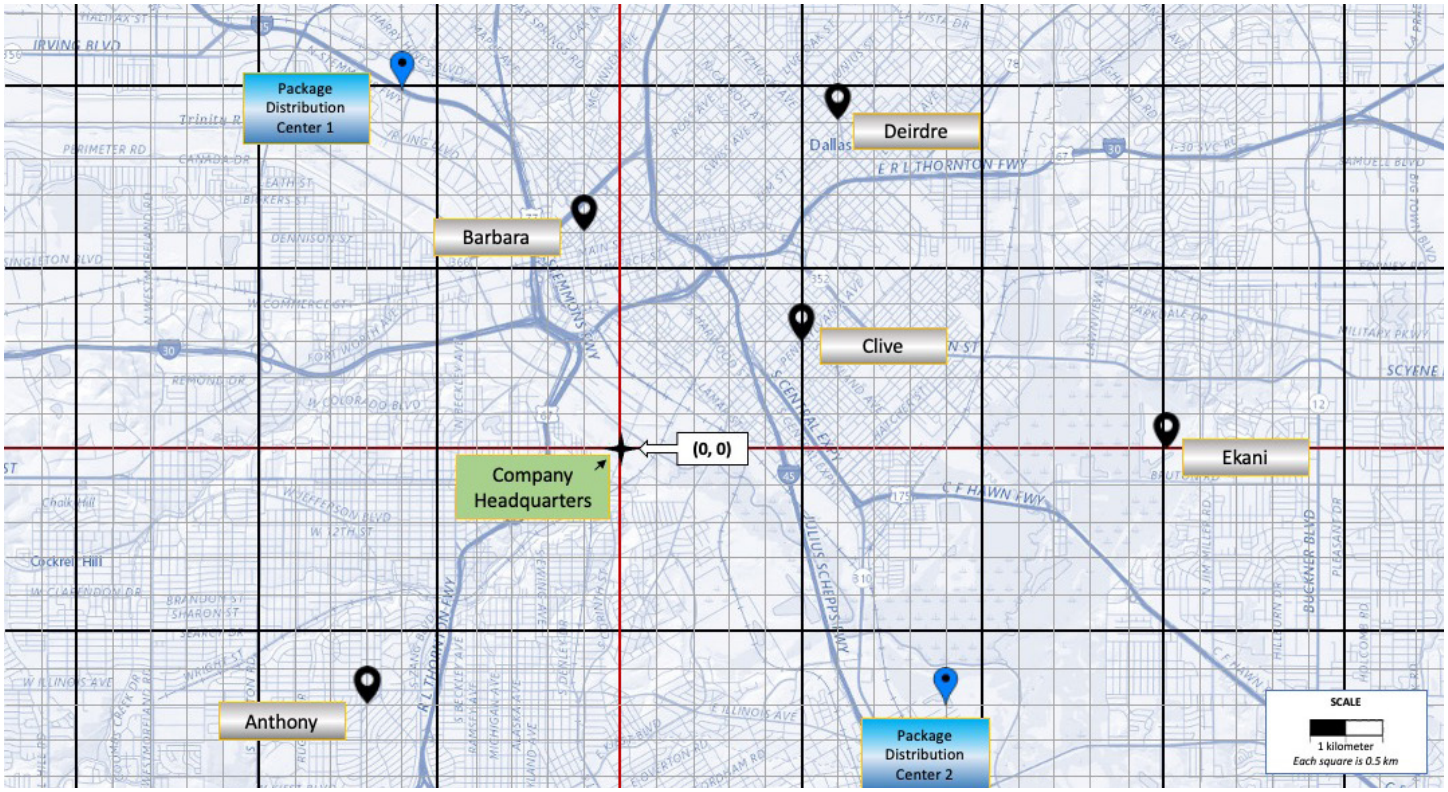

Figure 5. Grid with map for all quadrants. Credit: USGS

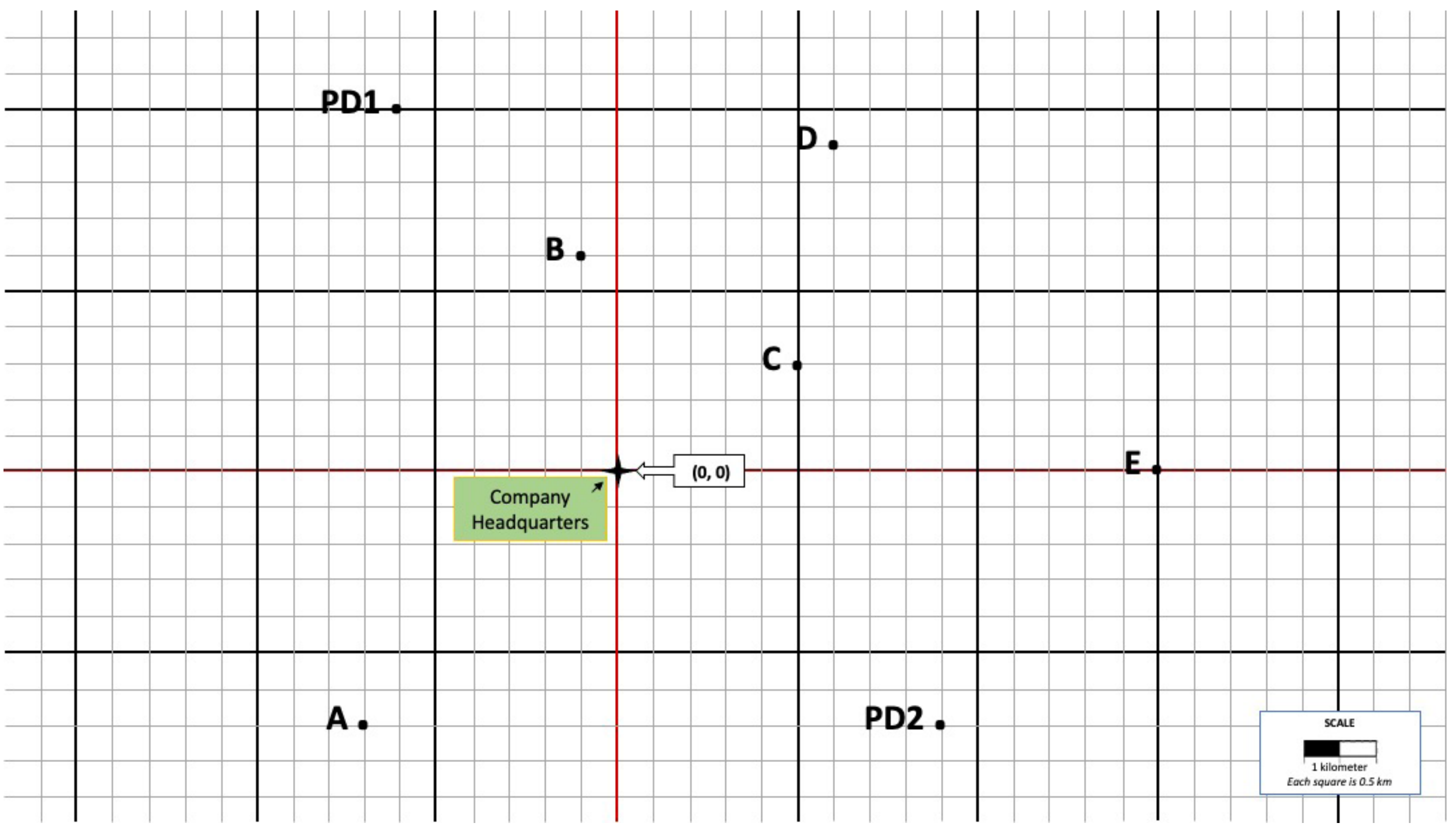

Figure 6. Grid without maps for all quadrants.

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