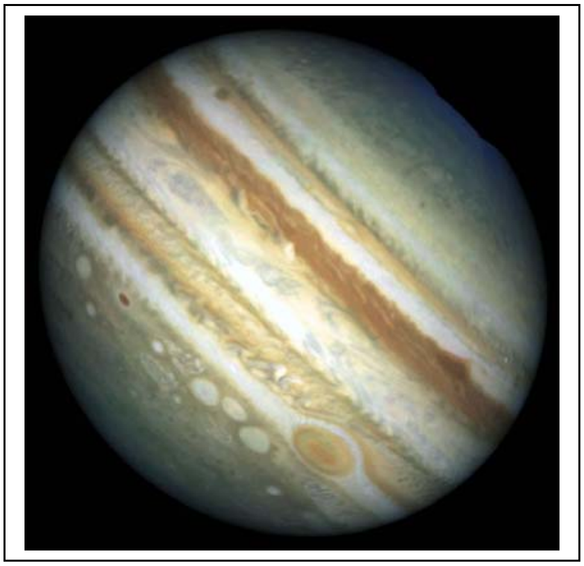
The Composition of Planetary Atmospheres



All of the planets in our solar system, and some of its smaller bodies too, have an outer layer of gas we call the atmosphere. The atmosphere usually sits atop a denser, rocky crust or planetary core. Atmospheres can extend thousands of kilometers into space.

The table below gives the name of the kind of gas found in each object's atmosphere, and the total mass of the atmosphere in kilograms. The table also gives the percentage of the atmosphere composed of the gas.

Object	Mass (kilograms)	Carbon Dioxide	Nitrogen	Oxygen	Argon	Methane	Sodium	Hydrogen	Helium	Other
Sun	3.0×10^{30}							71%	26%	3%
Mercury	1000			42%			22%	22%	6%	8%
Venus	4.8×10^{20}	96%	4%							
Earth	1.4×10^{21}		78%	21%	1%					<1%
Moon	100,000				70%		1%		29%	
Mars	2.5×10^{16}	95%	2.7%		1.6%					0.7%
Jupiter	1.9×10^{27}							89.8%	10.2%	
Saturn	5.4×10^{26}							96.3%	3.2%	0.5%
Titan	9.1×10^{18}		97%			2%				1%
Uranus	8.6×10^{25}					2.3%		82.5%	15.2%	
Neptune	1.0×10^{26}					1.0%		80%	19%	
Pluto	1.3×10^{14}	8%	90%			2%				

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth.

Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto.

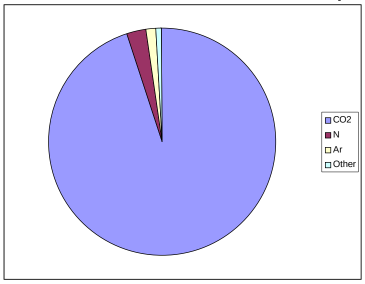
Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen?

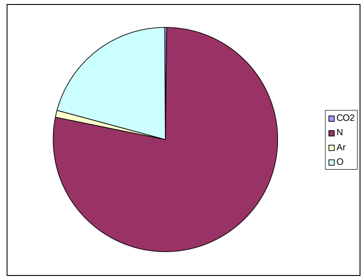
Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen?

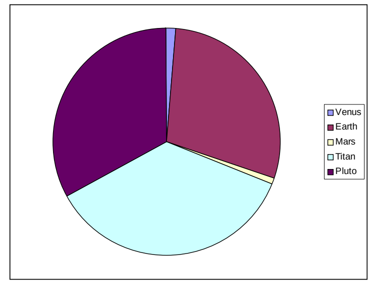
Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer Key

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth. Answer: Mars (left), Earth (middle)







Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto. Answer: First add up all the percentages for Nitrogen in the column to get 271.7%. Now divide each of the percentages in the column by 271.7% to get the percentage of nitrogen in the planetary atmospheres that is taken up by each of the planets: Venus = (4/271) = 1.5%; Earth = (78/271) = 28.8%, Mars = (2.7/271)= 1.0%, Titan = (97/271)=35.8%, Pluto=(90/271)=33.2%. Plot these new percentages in a pie graph (see above right). This pie graph shoes that across our solar system, Earth, Titan and Pluto have the largest percentage of nitrogen. In each case, the source of the nitrogen is from similar physical processes involving the chemistry of the gas methane (Titan and Earth) or methane ice (Pluto).

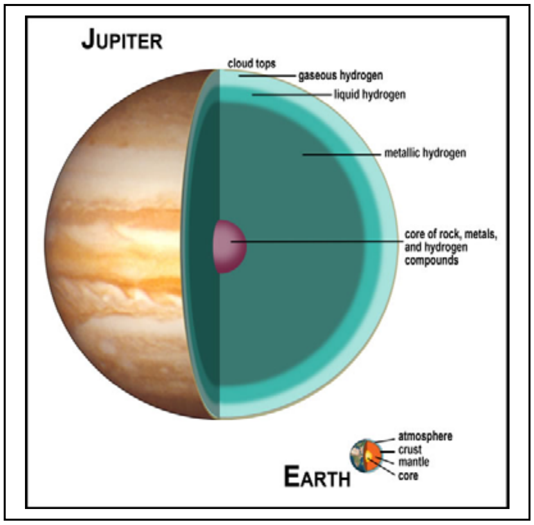
Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen? Answer: From the table we see that **Mercury** has the greatest percentage of oxygen in its atmosphere.

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen? Answer: Only two planets have detectable oxygen: Earth and Mercury. Though mercury has the highest percentage of oxygen making up its atmosphere, the number of kilograms of oxygen is only 1000 kg x 0.42 = 420 kilograms. By comparison, **Earth** has a smaller percentage of oxygen (21%) but a vastly higher quantity: 1.4×10^{21} kg x 0.21 = **2.9×10^{20} kilograms**. (That's 290,000,000,000,000,000,000 kg)

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer: The objects with the highest percentage of hydrogen are the sun, Mercury, Jupiter, Saturn, Uranus and Neptune. The objects with the least percentage are Venus, Earth, Moon, Mars, Titan, Pluto. With the exception of Mercury, which has a very thin atmosphere, the high-percentage objects are the largest bodies in the solar system. The planet Jupiter, Saturn, Uranus and Neptune are sometimes called the Gas Giants because so much of the mass of these planets consists of a gaseous atmosphere. These bodies generally lie far from the sun. The low-percentage objects are among the smallest bodies in the solar system. They are called the 'Rocky Planets' to emphasize their similarity in structure, where a rocky core and mantel are surrounded by a thin atmosphere. Most of these bodies lie close to the sun.

Measuring Atmospheric Trace Gases with Parts Per Million



Most of the planets in our solar system have two or three constituents that make up most of the atmosphere. For example, Venus and Mars have more than 98% of their atmosphere in carbon dioxide and nitrogen, while Earth has 99% of its atmosphere in nitrogen and oxygen. But trace gases with percentages below 1% are also important. For example, without the 0.3% of carbon dioxide in Earth's atmosphere, Earth would be a lifeless and frigid planet!

Scientists use 'parts per million' to represent the amounts of trace gases in a planetary atmosphere.

Examples: One year is 1 part per hundred of a century, or 1% of a century.
One year is 1 part per thousand of a millennium, or 0.1% of a millennium.
One millimeter is 1 part per million of 1 kilometer, or 0.0001% of a kilometer.

Problem 1 - The following list gives the percentages of various trace gases in the atmospheres of the indicated objects. Convert these percentages to parts-per-million (ppm) units.

Earth:	Carbon Dioxide………	0.038%..............	__________	ppm
	Neon………………....	0.00182%..........	__________	ppm
	Methane………………	0.000175%........	__________	ppm
	Water Vapor………...	5.0 %................	___________	ppm
Mars:	Neon………………....	0.00025%..........	___________	ppm
	Methane………………	0.00000105%.....	____________	ppm
Titan:	Methane………………	1.4%	____________	ppm
	Argon………………....	0.0043%.............	____________	ppm
	Carbon Monoxide…...	0.0052%.............	___________	ppm
	Ethane………………..	0.0013%.............	___________	ppm
Jupiter:	Methane……………..	0.3%...................	___________	ppm
	Ammonia…………….	0.026%...............	___________	ppm
	Ethane……………….	0.00058………….	___________	ppm
	Water Vapor…….....	0.0004……………	___________	ppm

Problem 2 - Using your ppm answers from Problem 1, by what factor does Earth have more methane than Mars as a trace gas?

Problem 3 – The amount of carbon dioxide in Earth's atmosphere is increasing by 2.5 ppm per year. If its value in 2012 was measured to be 392 ppm, by what year will it have reached 517 ppm? How old will you be then, if the growth rate continues at this rate of increase?

Problem 1 - The following list gives the percentages of various trace gases in the atmospheres of the indicated objects. Convert these percentages to parts-per-million (ppm) units.
Answer: Example: Earth CO2. 0.038% = 0.00038 then 0.00038 x 1,000,000 = 380 ppm

Earth:	Carbon Dioxide………	0.038%..............	**380**	**ppm**
	Neon………………....	0.00182%..........	**18.2**	**ppm**
	Methane………………	0.000175%........	**1.75**	**ppm**
	Water Vapor………....	5.0 %................	**50000**	**ppm**
Mars:	Neon………………....	0.00025%..........	**2.5**	**ppm**
	Methane………………	0.00000105%.....	**0.0105**	**ppm** or 10.5 ppb
Titan:	Methane………………	1.4%	**14000**	**ppm**
	Argon………………....	0.0043%.............	**43**	**ppm**
	Carbon Monoxide…...	0.0052%.............	**52**	**ppm**
	Ethane………………..	0.0013%.............	**13**	**ppm**
Jupiter:	Methane……………..	0.3%...................	**3000**	**ppm**
	Ammonia…………….	0.026%...............	**260**	**ppm**
	Ethane……………….	0.00058………….	**5.8**	**ppm**
	Water Vapor…….....	0.0004……………	**4.0**	**ppm**

Note. When the value for ppm is less than one, use parts per billion by multiplying ppm by 1000. Example, for Mars and Methane, 0.0105 ppm = 0.0105 x1000 = 10.5 ppb.

Problem 2 - Using your ppm answers, by what factor does Earth have more Methane than Mars as a trace gas?

Answer: Earth/Mars = 1.75 ppm/0.0105 ppm = **167 times more than Mars**.

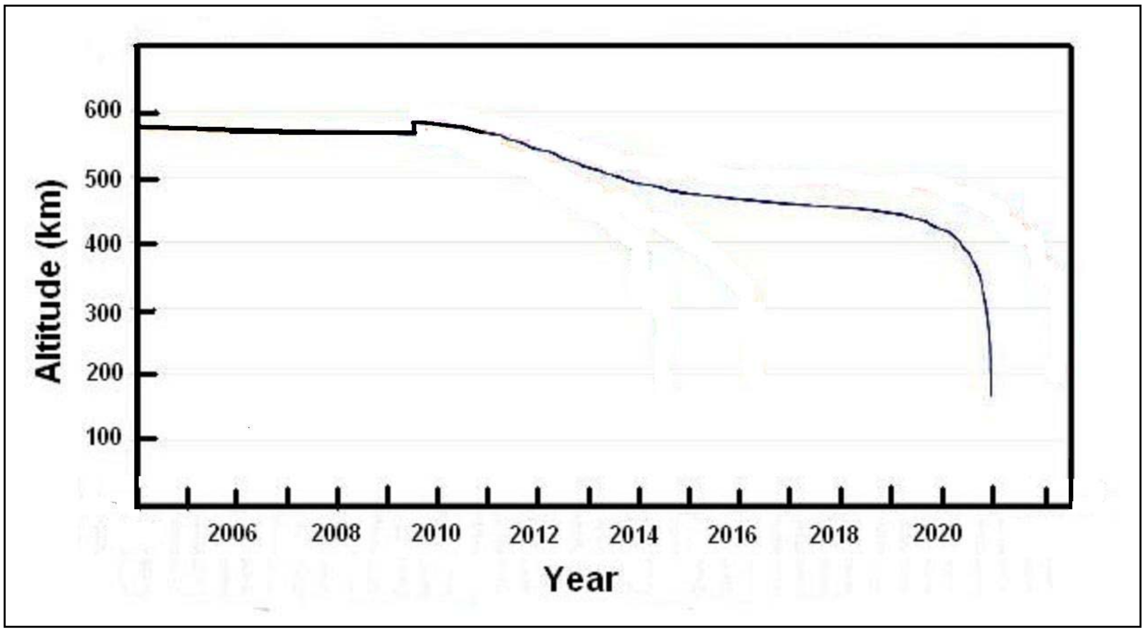
Problem 3 – The amount of carbon dioxide in Earth's atmosphere is increasing by 2.5 ppm per year. If its value in 2012 was measured to be 392 ppm, by what year will it have reached 517 ppm? How old will you be then, if the growth rate continues at this rate of increase?

Answer: To get to 517 ppm from 392 ppm it must increase by 517-392 = 125 ppm. If the increase is 2.5 ppm each year, it will take 125/2.5 = 50 years, so the year will be 2062.
If a student was 15 years old in 2012, they will be 65 years old in 2062.

Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, in May, 2009, no further Shuttle visits are planned. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry.

The curve below shows the predicted altitude for that last planned re-boost in 2009 of 18-km. NASA plans to use a robotic spacecraft after ca 2015 to allow a controlled re-entry for HST, but if that were not the case, it would re-enter the atmosphere sometime after 2020.



Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Answer: The Servicing Mission occurred in 2009. The upgrades and gyro repairs extend the satellite's operations by 5 more years, so if it re-enters after 2020 it will have about 6 years to go before uncontrolled re-entry.

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Answer: Use a millimeter ruler to determine the scale of the horizontal axis in weeks per millimeter. Mark the point on the curve that corresponds to a vertical value of 400 km. Draw a line to the horizontal axis and measure its distance from 2013 in millimeters. Convert this to weeks using the scale factor you calculated. The answer should be about 50 weeks.

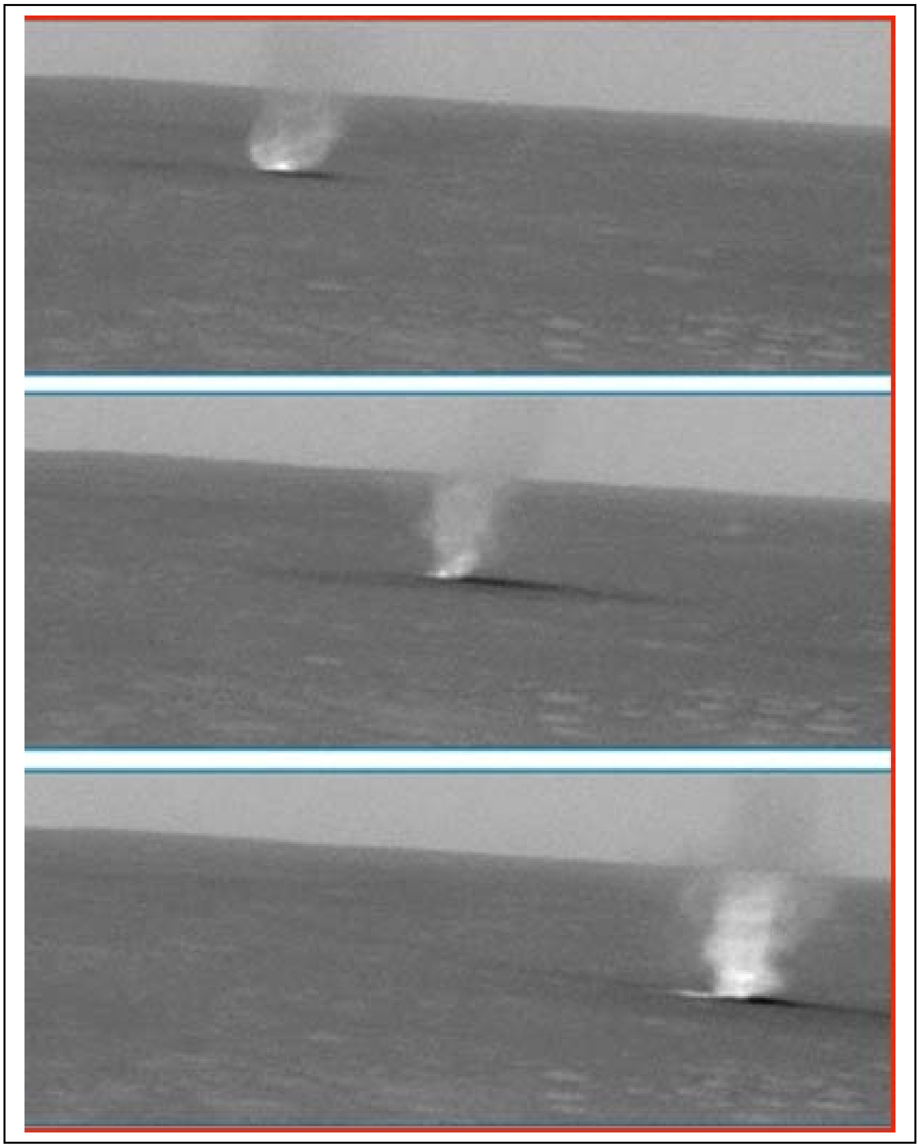
"NASA's 23-year-old Hubble Space Telescope is still going strong, and agency officials said Tuesday (Jan. 8, 2013) they plan to operate it until its instruments finally give out, potentially for another six years at least.

After its final overhaul in 2009, the Hubble telescope was expected to last until at least 2015. Now, NASA officials say they are committed to keeping the iconic space observatory going as long as possible.

"Hubble will continue to operate as long as its systems are running well," Paul Hertz, director of the Astrophysics Division in NASA's Science Mission Directorate, said here at the 221st meeting of the American Astronomical Society. Hubble, like other long-running NASA missions such as the Spitzer Space Telescope, will be reviewed every two years to ensure that the mission is continuing to provide science worth the cost of operating it, Hertz added."

(Space.com, January 9, 2013.)

The Martian Dust Devils



A dust devil spins across the surface of Gusev Crater just before noon on Mars. NASA's Spirit rover took the series of images to the left with its navigation camera on March 15, 2005.

The images were taken at:

11:48:00 (T=top)
11:49:00 (M=middle)
11:49:40 (B=bottom)

based upon local Mars time.

The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

A simple application of the rate formula

$$speed = \frac{\text{distance}}{\text{time}}$$

lets us estimate how fast the dust devil was moving.

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

Problem 2 - What was the time difference, in seconds, between the images T-M, M-B and T-B?

Problem 3 - What was the distance, in meters, traveled between the images T-M and M-B?

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B. If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B?

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B?

Answer Key

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top and bottom frames? Answer: The location of the dust devil in frame B when placed in image T is a shift of about 65 millimeters, which at a scale of 7.4 meters/mm equals about 480 meters.

Problem 2 - What was the time difference between the images T-M, M-B and T-B? Answer: T-M = 11:49:00 - 11:48:00 - 1 minute or 60 seconds. For M-B the time interval is 40 seconds. For T-B the time interval is 100 seconds.

Problem 3 - What was the distance traveled between the images T-M and M-B? Answer: T-M = about 30 mm or 222 meters; M-B = about 35 mm or 259 meters.

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B? If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?
Answer: Speed = distance/time so 480 meters/100 seconds = 4.8 meters/sec. This is about twice as fast as an astronaut can walk, so running would be a better option.

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B? Answer: Speed(T-M) = 222 meters/60 seconds = 3.7 meters/sec. Speed(M-B) = 259 meters/40 seconds = 6.5 meters/sec.

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B? Answer: because the speed increased from 4.8 meters/sec to 6.5 meters/sec, the dust

The Changing Pace of Global Warming

Table of Global Temperature Anomalies

Year	Temperature (degrees C)	Year	Temperature (degrees C)
1900	-0.20	1960	+0.05
1910	-0.35	1970	0.00
1920	-0.25	1980	+0.20
1930	-0.28	1990	+0.30
1940	+0.08	2000	+0.45
1950	-0.05	2010	+0.63

A new study by researchers at the Goddard Institute for Space Studies determined that 2010 tied with 2005 as the warmest year on record, and was part of the warmest decade on record since the 1800s. The analysis used data from over 1000 stations around the world, satellite observations, and ocean and polar measurements to draw this conclusion.

The table above gives the average 'temperature anomaly' for each decade from 1900 to 2010. The Temperature Anomaly is a measure of how much the global temperature differed from the average global temperature between 1951 to 1980. For example, a +1.0 C temperature anomaly in 2000 means that the world was +1.0 degree Celsius warmer in 2000 than the average global temperature between 1951-1980.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000?

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/La Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature anomaly change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

Problem 3 - From the data in the table, calculate the rate of change of the temperature anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900?

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000? Answer: In 1900 it was -0.20 C and in 2000 it was +0.45, so it has changed by +0.45 - (-0.20) = **+0.65 C**.

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/El Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.Answer:

1900 to 1920: -0.25C - (-0.20C) = **-0.05 C a decrease (cooling) of 0.05 C**

1920 to 1950: - 0.05C - (-0.25C) = **+0.20 C an increase (warming) of 0.20 C**

1950 to 1980: +0.20C - (-0.05C) = **+0.25 C an increase (warming) of 0.25 C**

1980 to 2010: +0.63C - (+0.20C) = **+0.43 C an increase (warming) of 0.43 C**

Problem 3 - From the data in the table, calculate the rate of change of the Temperature Anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900? Answer:

1900 to 1920: -0.05 C/3 decades = **-0.017 C per decade**

1920 to 1950: +0.20 C/ 3 decades = **+0.067 C per decade**

1950 to 1980: +0.25 C/3 decades = **+0.083 C per decade**

1980 to 2010: +0.43 C/3 decades = **+0.143 C per decade**.

The pace of global temperature change is **increasing in time**. It is almost doubling every 10 years.

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Answer: Students may graph the data in the table, then use a ruler to draw a line on the graph between 1960 and 2000, to extrapolate to the temperature anomaly in 2050. A linear equation, T = mx + b, that models this data is b = +0.05C m = (+0.45 - 0.05)/4 decades so m = +0.10 C/decade. Then T = +0.10x + 0.05. For 2050, which is 9 decades after 1960, x=9 so T = +0.1(9) + 0.05 = **+0.95 C**. So, the world will be, on average, about **+1 C warmer** in 2050 compared to its average temperature between 1950 and 1980. This assumes a linear change in T with time.

However from Problem 3 we see that the temperature anomaly change is accelerating. The 'second order' differences are +0.033, +0.033, +0.06. If we take the average change as (0.033+0.033+0.06)/3 = +0.042 we get a more accurate 'quadratic' expression: T = $+0.042 x^2 +0.1x + 0.05$. For the year 2050, this quadratic prediction suggests T = $0.042(9)^2 +0.1(9) + 0.05$ so T = +1.32 C.

For more information about this research, see the NASA Press Release at **http://www.nasa.gov/topics/earth/features/2010-warmest-year.html**

A very common way to describe the atmosphere of a planet is by its 'scale height'. This quantity represents the vertical distance above the surface at which the density or pressure if the atmosphere decreases by exactly 1/e or $(2.718)^{-1}$ times (equal to 0.368).

The scale height, usually represented by the variable **H**, depends on the strength of the planet's gravity field, the temperature of the gases in the atmosphere, and the masses of the individual atoms in the atmosphere. The equation to the left shows how all of these factors are related in a simple atmosphere model for the density P. The variables are:

$$P(z) = P_0\, e^{-\frac{z}{H}} \quad \text{and} \quad H = \frac{kT}{mg}$$

z: Vertical altitude in meters
T: Temperature in Kelvin degrees
m: Average mass of atoms in kilograms
g: Acceleration of gravity in meters/sec^2
k: Boltzmann's Constant 1.38×10^{-23} J/deg

Problem 1 - For Earth, g = 9.81 meters/sec^2, T = 290 K. The atmosphere consists of 22% O_2 (m= $2\times2.67\times10^{-26}$ kg) and 78% N_2 (m= $2\times2.3\times10^{-26}$ kg). What is the scale height, H?

Problem 2 - Mars has an atmosphere of nearly 100% CO_2 (m=7.3×10^{-26} kg) at a temperature of about 210 Kelvins. What is the scale height H if g= 3.7 meters/sec^2?

Problem 3 - The Moon has an atmosphere that includes about 0.1% sodium (m=6.6×10^{-26} kg). If the scale height deduced from satellite observations is 120 kilometers, what is the temperature of the atmosphere if g= 1.6 meters/sec^2?

Problem 4 - At what altitude on Earth would the density of the atmosphere P(z) be only 10% what it is at sea level, P_0?

Problem 5 - Calculate the total mass of the atmosphere in a column of air, below a height h with integral calculus. At what altitude, h, on Earth is half the atmosphere below you?

Answer Key

Problem 1 - Answer: First we have to calculate the average atomic mass. <m> = 0.22 ($2 \times 2.67 \times 10^{-26}$ kg) + 0.78 ($2 \times 2.3 \times 10^{-26}$ kg) = 4.76×10^{-26} kg. Then,

$$H = \frac{(1.38 \times 10^{-23})(290)}{(4.76 \times 10^{-26})(9.81)} =$$ **8,570 meters or about 8.6 kilometers.**

Problem 2 - Answer:

$$H = \frac{(1.38 \times 10^{-23})(210)}{(7.3 \times 10^{-26})(3.7)} =$$ **10,700 meters or about 10.7 kilometers.**

Problem 3 - Answer

$$T = \frac{(6.6 \times 10^{-26})(1.6)(120000)}{(1.38 \times 10^{-23})} =$$ **918 Kelvins.**

Problem 4 - Answer: $0.1 = e^{-(z/H)}$, Take ln of both sides, $\ln(0.1) = -z/H$ then $z = 2.3\,H$ so for H= 8.6 km, **z = 19.8 kilometers**.

Problem 5 - First calculate the total mass:

$$M = \int_0^\infty P(z)dz \qquad M = P_0 \int_0^\infty e^{-\frac{z}{H}} dz \qquad M = P_0 H \int_0^\infty e^{-x} dx \qquad M = P_0 H$$

Then subtract the portion above you:

$$m = \int_h^\infty P(z)dz \qquad M = P_0 \int_h^\infty e^{-\frac{z}{H}} dz \qquad M = P_0 H \int_h^\infty e^{-x} dx \qquad M = P_0 H \left[e^{-h} - 1 \right]$$

To get: $dm(h) = P_0 H e^{-h}$

So $1/2 = e^{-h/H}$ and $h = \ln(2)(8.6 \text{ km})$ and so the height is **6 kilometers!**

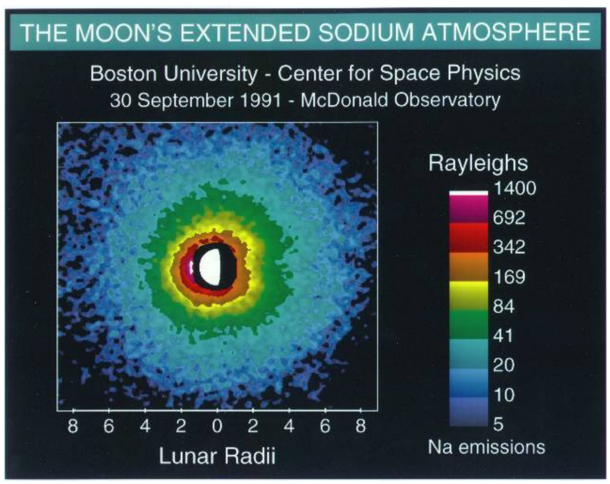
The Moon's Atmosphere!

Courtesy: T.A.Rector, I.P.Dell'Antonio (NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6×10^{-24} grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

Answer Key:

Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6 x 10-24 grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen = 1.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 6.4 x 10^{-20} grams

B) Helium = 4.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.6 x 10^{-19} grams

C) Neon = 20.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 1.3 x 10^{-18} grams

D) Argon = 36.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.3 x 10^{-18} grams

E) Total = (0.064 + 0.26 + 1.3 + 2.3) x 10^{-18} grams = <u>3.9 x 10^{-18} grams per cc.</u>

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of Ri = 1,738 km and Ro = 1,738+170 = 1,908 km. V = $4/3\ \pi\ (1908)^3 - 4/3\pi\ (1738)^3 = 2.909 \times 10^{10}\ km^3 - 2.198 \times 10^{10}\ km^3 = 7.1 \times 10^9\ km^3$

Volume = $7.1 \times 10^9\ km^3$ **x** (10^5 cm/km) **x** (10^5 cm/km) **x** (10^5 cm/km)

= $7.1 \times 10^{24}\ cm^3$

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of Ri, multiplied by the shell thickness of h = 170 km, you will get a slightly different answer of $6.5 \times 10^9\ km^3$ and $6.5 \times 10^{24}\ cm^3$

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

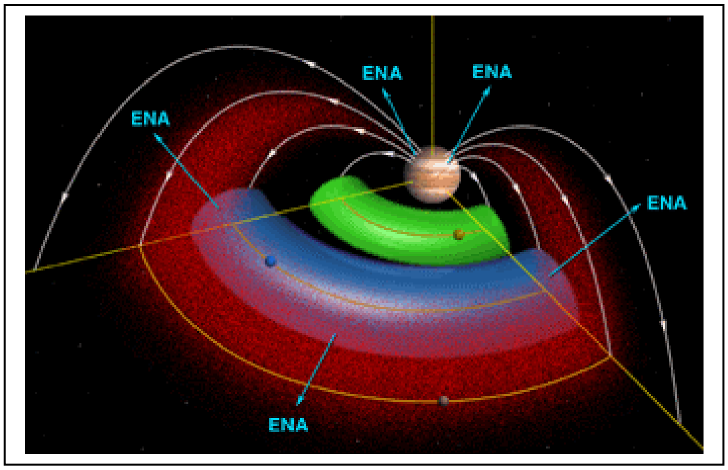
A) Mass = density x volume = (3.9 x 10^{-18} gm/cc) x 7.1 x 10^{24} cm^3 = 2.8 x 10^7 grams

B) Mass = 2.8 x 10^7 grams x (1 kg/1000 gms) = 28,000 kilograms.

C) Mass = 28,000 kg x (1 ton / 1000 kg) = 28 tons.

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.

The Io Plasma Torus



The satellite of Jupiter, Io, is a volcanically active moon that ejects 1,000 kilograms of ionized gas into space every second. This gas forms a torus encircling Jupiter along the orbit of Io. We will estimate the total mass of this gas based on data from the NASA Cassini and Galileo spacecraft.

Image: Io plasma torus (Courtesy NASA/Cassini)

Problem 1 - Galileo measurements obtained in 2001 indicated that the density of neutral sodium atoms in the torus is about 35 atoms/cm^3. The spacecraft also determined that the inner boundary of the torus is at about 5 Rj, while the outer boundary is at about 8 Rj. (1 Rj = 71,300 km). A torus is defined by the radius of the ring from its center, R, and the radius of the circular cross section through the donut, r. What are the dimensions, in kilometers, of the Io torus based on the information provided by Galileo?

Problem 2 - Think of a torus as a curled up cylinder. What is the general formula for the volume of a torus with radii R and r?

Problem 3 - From the dimensions of the Io torus, what is the volume of the Io torus in cubic meters?

Problem 4 - From the density of sodium atoms in the torus, what is A) the total number of sodium atoms in the torus? B) If the mass of a sodium atom is 3.7 x 10^{-20} kilograms, what is the total mass of the Io torus in metric tons?

Calculus:

Problem 5 - Using the 'washer method' in integral calculus, derive the formula for the volume of a torus with a radius equal to R, and a cross-section defined by the formula $x^2 + y^2 = r^2$. The torus is formed by revolving the cross section about the Y axis.

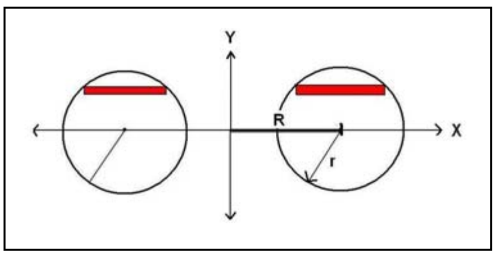
Problem 1 - The mid point between 5 Rj and 8 Rj is (8+5)/2 = 6.5 Rj so R = 6.5 Rj and r = 1.5 Rj. Then R = 6.5 x 71,300 so **R = 4.6 x 10^5 km**, and r = 1.5 x 71,300 so **r = 1.1 x 10^5 km**.

Problem 2 - The cross-section of the cylinder is πr^2, and the height of the cylinder is the circumference of the torus which equals $2 \pi R$, so the volume is just V = (2π R) x (πr^2) or **$V = 2 \pi^2 R r^2$.**

Problem 3 - Volume = $2 \pi^2$ (4.6 x 10^5 km) (1.1 x 10^5 km $)^2$ so **V = 1.1 x 10^{17} km^3.**

Problem 4 - A) 35 atoms/cm^3 x (100000 cm/1 km$)^3$ = 3.5 x 10^{16} atoms/km^3. Then number = density x volume so N = (3.5 x 10^{16} atoms/km^3) x (1.1 x 10^{17} km^3) , so **N = 3.9 x 10^{33} atoms**. B) The total mass is M = 3.9 x 10^{33} atoms x 3.7 x 10^{-20} kilograms/atom = 1.4 x 10^{14} kilograms. 1 metric ton = 1000 kilograms, so the total mass is **M = 100 billion tons**.

Advanced Math:



$$V = 8 \pi R \int_0^r \left(r^2 - y^2 \right)^{1/2} dy$$

Recall that the volume of a washer is given by V = π (R(outer$)^2$ - R(inner$)^2$) x thickness. For the torus figure above, we see that the thickness is just dy. The distance from the center of the cross section to a point on the circumference is given by $r^2 = x^2 + y^2$. The width of the washer (the red volume element in the figure) is parallel to the X-axis, so we want to express its length in terms of y, so we get $x = (r^2 - y^2)^{1/2}$. The location of the outer radius is then given by R(outer) = R + $(r^2-y^2)^{1/2}$, and the inner radius by R(inner) = R - $(r^2 - y^2)^{1/2}$. We can now express the differential volume element of the washer by dV = π [$(R + (r^2-y^2)^{1/2})^2 - (R - (r^2 - y^2)^{1/2})^2$] dy. This simplifies to dV = π [4R $(r^2-y^2)^{1/2}$] dy or dV = $4 \pi R (r^2-y^2)^{1/2}$ dy. The integral can immediately be formed from this ,with the limits y = 0 to y=r. Because the limits to y only span the upper half plane, we have to double this integral to get the additional volume in the lower half-plane. The required integral is shown above.

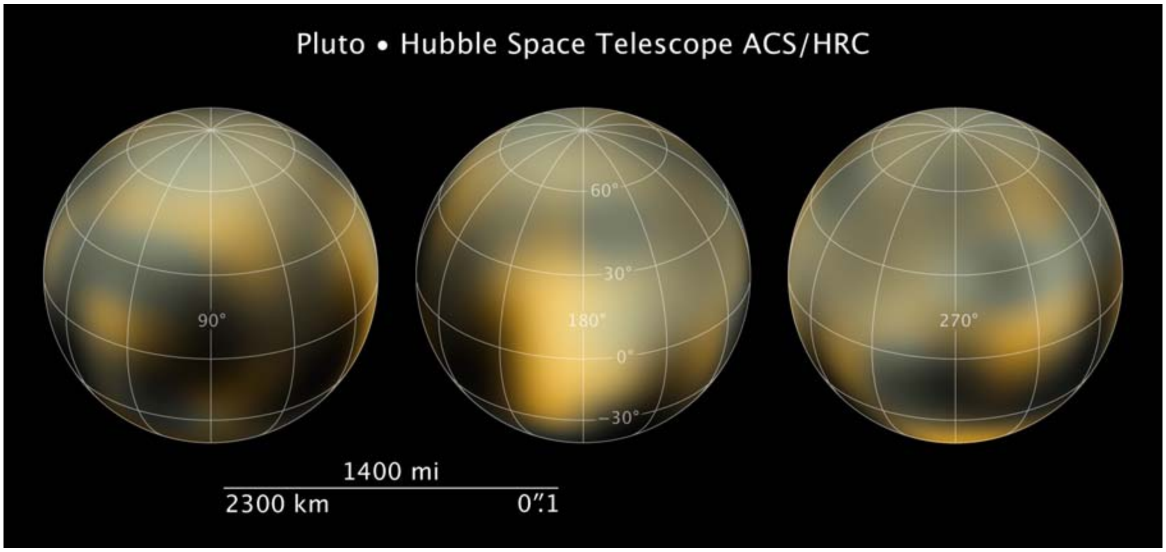
This integral can be solved by factoring out the r from within the square-root, then using the substitution U = y/r and dU = 1/r dy to get the integrand dV = $8 \pi R r^2 (1 - U^2)^{1/2}$ dU. The integration limits now become U=0 to U=1. Since r and R are constants, this is an elementary integral with the solution V = 1/2 U $(1-U^2)^{1/2}$ + 1/2 arcsin(U). When this is evaluated from U=0 to U=1, we get

$V = 8 \pi R r^2$ [0 + 1/2 arcsin(1)] - [0 + 1/2arcsin(0)]

$V = 8 \pi R r^2$ 1/2 (π/2)

$V = 2 \pi R r^2$

The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = 1.5×10^{11} meters).

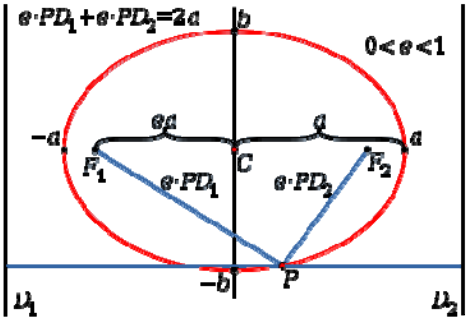
Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R) = \left(\frac{L(1-A)}{16\pi\sigma R^2}\right)^{\frac{1}{4}} \quad \text{Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); σ is a constant with a value of 5.67×10^{-8}, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

Problem 1 - Answer:



In Standard Form $2433600=1521x^2+1600y^2$ becomes $1=\frac{x^2}{1600}+\frac{y^2}{1521} = \frac{x^2}{a^2}+\frac{y^2}{b^2}$

Then A) **a = 40 AU** and B) **b=39 AU**. C) The ellipticity $\mathbf{e = (a^2 - b^2)^{1/2}/a = 0.22}$. D) The longest distance from a focus is just a(1 + e) = 40(1+0.22) = **49 AU**. E) The shortest distance is just a(1-e) = (1-0.22)(40) = **31 AU**. Written out in meters we have a= 6×10^{12} meters; b= 5.8×10^{12} meters; aphelion = 7.35×10^{12} meters and perihelion = 4.6×10^{12} meters.

Problem 2 - Answer: For R in terms of AU, the formula simplifies to

$$T(R)=\left(\frac{4x10^{26}(1-0.6)}{16(3.14)(5.67x10^{-8})(1.5x10^{11})^2R^2}\right)^{\frac{1}{4}} \quad \text{so} \quad T(R)=\frac{223}{\sqrt{R}} \text{ K}$$

A) For a perihelion distance of 31 AU we have T = $223/(31)^{1/2}$ = 40 K; B) At an aphelion distance of 49 AU we have T = $223/(49)^{1/2}$ = 32 K. Note: The actual temperatures are about higher than this and average about 50K.

Problem 3 - Answer: At aphelion, the height of the atmosphere is about H=1.2x(32) = 38 kilometers, and at perihelion it is about H=1.2x(40) = 48 kilometers, so as Pluto orbits the sun its atmosphere increases and decreases in thickness.

Note: In fact, because the freezing point of methane is 91K, at aphelion most of the atmosphere freezes onto the surface of the dwarf planet, and at aphelion it returns to a mostly gaseous state. This indicates that the simple physical model used to derive H(T) was incomplete and did not account for the freezing-out of an atmospheric constituent.