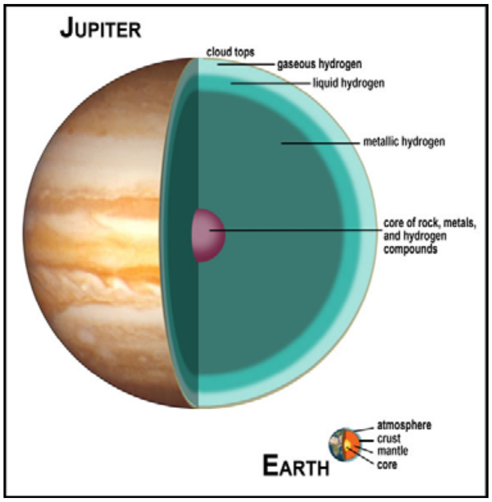
Relative Sizes of the Planets and other Objects



The actual sizes of the major objects in our solar system range from the massive planet Jupiter, to many small moons and asteroids no more than a few kilometers across.

It is often helpful to create a scaled model of the major objects so that you can better appreciate just how large or small they are compared to our Earth.

This exercise will let you work with simple proportions and fractions to create a scaled-model solar system.

Problem 1 - Jupiter is 7/6 the diameter of Saturn, and Saturn is 5/2 the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter?

Problem 2 – Earth is 13/50 the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn?

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km),Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters?

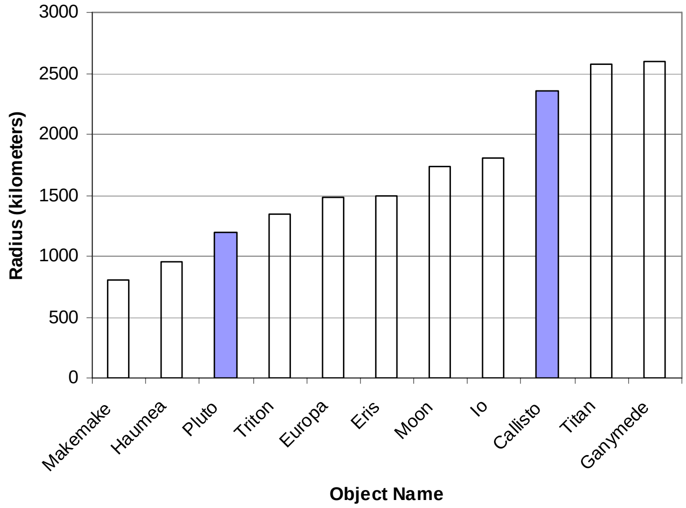
Answer Key

Problem 1 - Jupiter is 7/6 the diameter of Saturn, and Saturn is 5/2 the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter? Answer: 2/5 x 6/7 = **12/35**.

Problem 2 – Earth is 13/50 the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn? Answer: 5/2 x 50/13 = 250/26 = **125/13** times.

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km),Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

Answer: Average = (1738+1810+1500+1480+2600+2360+800+2575+1350+1200+950)/11 so **Average radius = 1669 km**. **Median radius = 1500 km**.

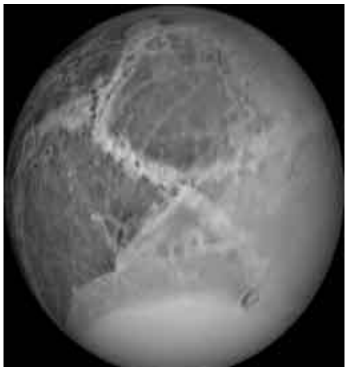


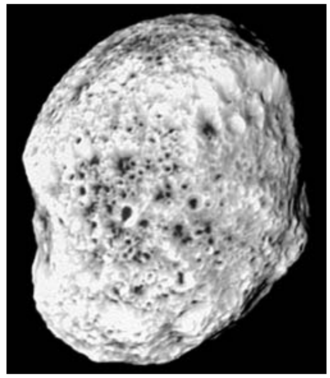
Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters? Answer: The scaled disk diameters are shown in Column 3 in millimeters.

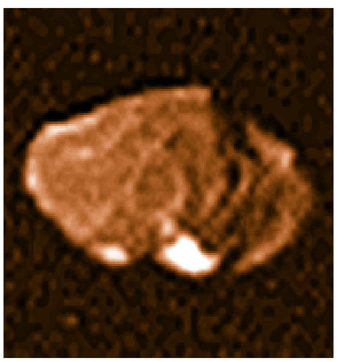
Object	Radius (km)	Diameter (mm)
Makemake	800	16
Haumea	950	20
Pluto	1200	25
Triton	1350	28
Europa	1480	30
Eris	1500	30
Moon	1738	36
Io	1810	38
Callisto	2360	48
Titan	2575	54
Ganymede	2600	54

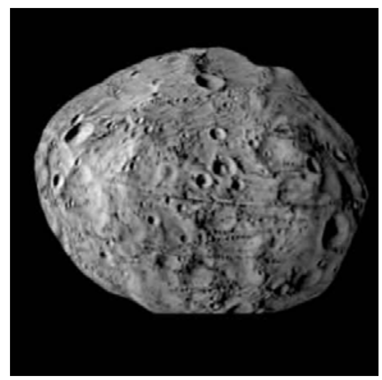
Astronomy in the Round

Why are many astronomical bodies round? Here is an activity in which you use astronomical photographs of various solar system bodies, and determine how big a body has to be before it starts to look round. Can you figure out what it is that makes a body round?









The images show the shapes of various astronomical bodies, and their sizes: Dione (560 km), Hyperion (205 x 130 km), Tethys (530 km), Amalthea (130 x 85 km), Ida (56 x 24 km), Phobos (14 x 11 km).

Question 1) How would you define the roundness of a body?

Question 2) How would you use your definition of roundness to order these objects from less round to round?

Question 3) Can you create from your definition a number that represents the roundness of the object?

Question 4) On a plot, can you compare the number you defined in Question 3 with the average size of the body?

Question 5) Can you use your plot to estimate the minimum size that a body has to be in order for it to be noticeably round? Does it depend on whether the body is mostly made of ice, or mostly of rock?

The images show the shapes of various astronomical bodies, and their sizes: Dione (560 km), Hyperion (205 x 130 km), Tethys (530 km), Amalthea (130 x 85 km), Ida (56 x 24 km), Phobos (14 x 11 km).

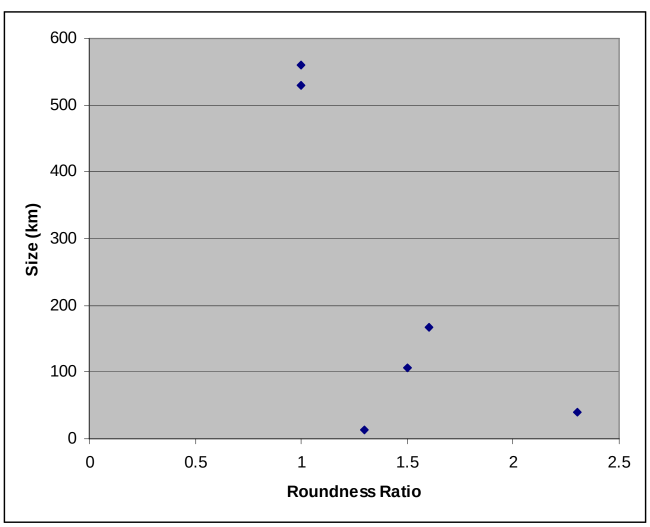
Question 1) How would you define the roundness of a body? **Answer: Students may explore such possibilities as the difference between the longest and shortest dimension of the object; the ratio of the longest to the shortest diameter; or other numerical possibilities.**

Question 2) How would you use your definition of roundness to order these objects from less round to round? **Answer. The order should look something like Ida, Amalthea, Hyperion, Phobos, Tethys and Dione.**

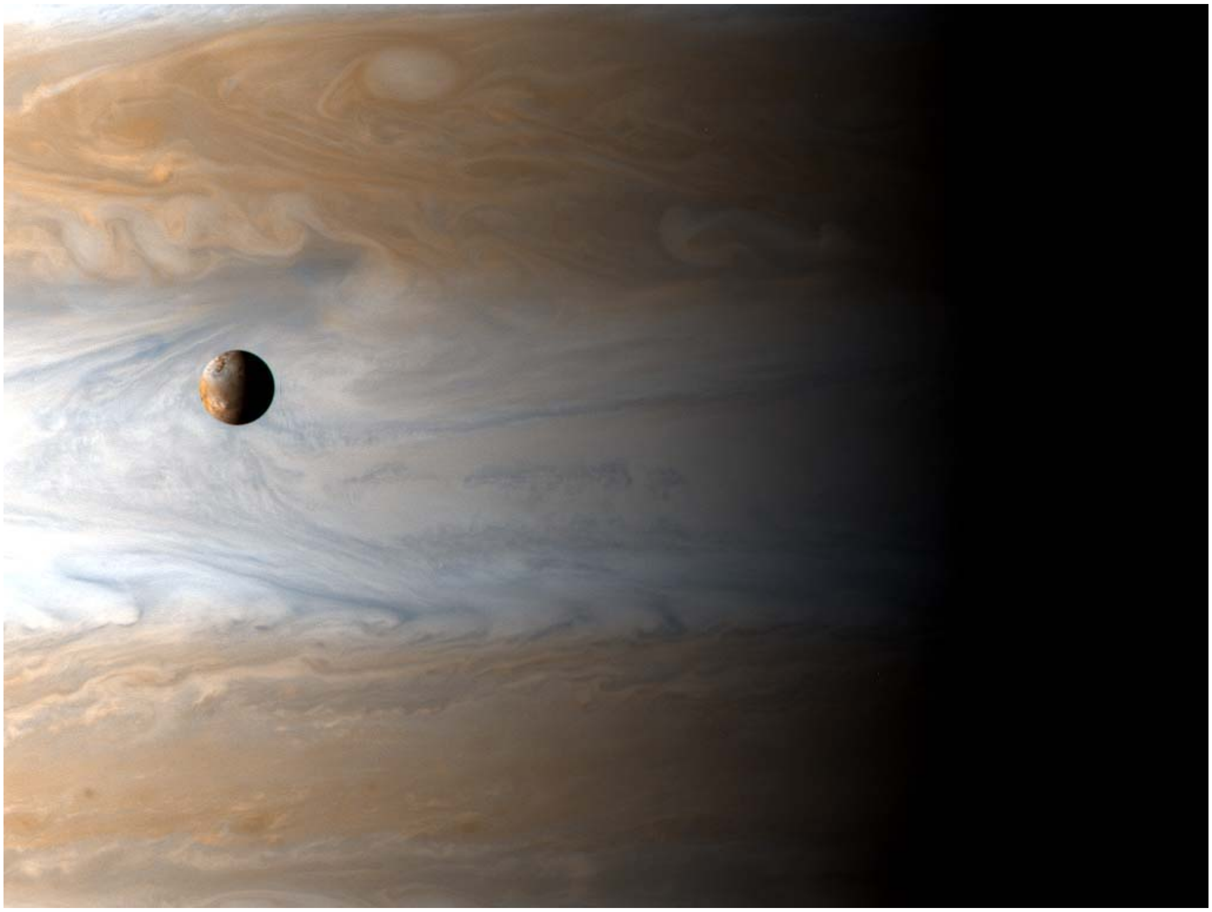
Question 3) Can you create from your definition a number that represents the roundness of the object? **Answer: If we select the ratio of the major to minor axis length, for example, we would get Dione = 1.0; Hyperion = 1.6; Tethys=1.0; Amalthea=1.5; Ida = 2.3 and Phobos = 1.3**

Question 4) On a plot, can you compare the number you defined in Question 3 with the average size of the body? **Answer: See plot below for the numerical definition selected in Question 3. We have used the average size of each irregular body defined as (L + S)/2. so Hyperion = (205+130)/2 = 167 km; Amalthea = (130+85)/2 = 107 km; Ida = (56+24)/2 = 40 km; Phobos = (14+11)/2 = 13 km.**

Question 5) Can you use your plot to estimate the minimum size that a body has to be in order for it to be noticeably round? Does it depend on whether the body is mostly made of ice, or mostly of rock? Answer: By connecting a smooth curve through the points (you can do this by eye), the data suggests that a body becomes noticeably round when it is at a size between 200-400 km. Students can obtain pictures of other bodies in the solar system and see if they can fill-in the plot better with small moons of the outer planets, asteroids (Ceres, etc) or even comet nuclei. Note, inner solar system bodies are mostly rock. Outer solar system bodies are mostly ice, so students might notice that by labeling the points as ‘rocky bodies’ or ‘icy bodies’ that they may see two different trends because ice is more pliable than rock. Students should investigate what the size (mass) has to do with roundness, and see that larger bodies have more gravity to deform their substance with.



Jupiter and Io



This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?
Question 4: What is the size of the smallest feature on Io that you can see?

Answer Key

This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?
Answer: 10 mm

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 3,600 km

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.
Answer: 3600 km / 10 mm = 360 km/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Answer: 160 mm x 119 mm = 58,000 km x 19,000 km

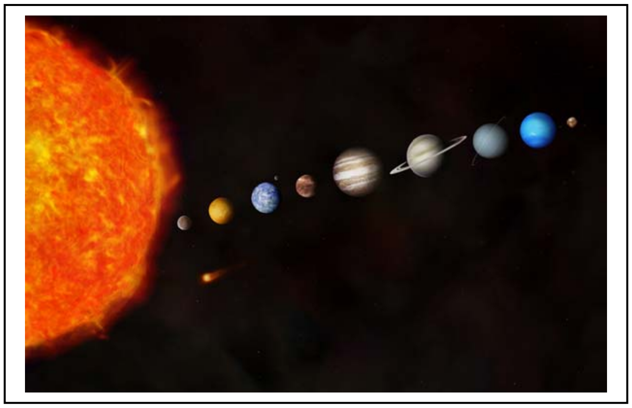
Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
Answer: The width of the white equatorial band is 45 mm or 16,000 km

Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?
Answer: The faint cloud streaks are 0.5 mm wide or 200 km across to one significant figure.

Question 4: What is the size of the smallest feature on Io that you can see?
Answer: The white spots in the southern hemisphere are 0.5 mm across or 200 km to one significant figure. This is a good time to mention that some details in an image can be artifacts from the printing process or defects in the camera itself. Students may find photocopying artifacts at 0.5 mm or less.

Note to teachers: The correct scale for Io and Jupiter will be slightly different depending on how far away the camera was when taking the picture. If the camera was very close to Io, then the scale you will infer for Io will be very different than for the more distant Jupiter because Io will take up more of the field-of-view in the image. Geometrically, for a fixed angle of separation between features on Io, this angle will subtend a SMALLER number of kilometers than the same angle on the more-distant Jupiter. However, if the distance from the camera to Jupiter/Io is very large, then as seen from the camera, both objects are at essentially the same distance and so there will be little difference between the scales used for the two bodies. Students can check this result with an inquiry assignment.

Planet Fractions and Scales



Some of the planets in our solar system are much bigger than Earth while others are smaller. By using simple fractions, you will explore how their sizes compare to each other.

Image courtesy NASA/Chandra Observatory/SAO

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is 1/4 the size of Neptune. How much larger is Saturn than Neptune?

Problem 2 - Earth is twice as big as Mars, but only 1/11 the size of Jupiter. How large is Jupiter compared to Mars?

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

Problem 4 - Mercury is 3/4 the size of Mars. How large is Earth compared to Mercury?

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

Problem 6 - The satellite of Saturn, called Titan, is 1/10 the size of Uranus. How large is Titan compared to Earth?

Problem 7 - The satellite of Jupiter, called Ganymede, is 2/5 the size of Earth. How large is it compared to Jupiter?

Problem 8 - The Dwarf Planet Pluto is 1/3 the diameter of Mars. How large is the diameter of Jupiter compared to Pluto?

Problem 9 - If the diameter of Earth is 13,000 km ,what are the diameters of all the other bodies?

Answer Key

Note to teachers: The actual diameters of the planets, in kilometers, are as follows

Mercury	4,900 km	Jupiter	143,000 km
Venus	12,000 km	Saturn	120,000 km
Earth	13,000 km	Uranus	51,000 km
Mars	6,800 km	Neptune	50,000 km

Also: Titan = 5,100 km, Ganymede = 5,300 km Ceres = 950 km, and Pluto 2,300 km

Advanced students (Grades 4 and above) may use actual planetary size ratios as decimal numbers, but for this simplified version (Grades 2 and 3), we approximate the size ratios to the nearest simple fractions. **Students may also use the information in these problems to make a scale model of the solar system in terms of the relative planetary sizes.**

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is 1/4 the size of Neptune. How much larger is Saturn than Neptune?
Answer: Neptune is 4x Venus and Saturn is 10x Venus, so Saturn is 10/4 = 5/2 times as big as Neptune.

Problem 2 - Earth is twice as big as Mars, but only 1/11 the size of Jupiter. How large is Jupiter compared to Mars?
Answer: Jupiter is 11 x Earth, and Mars is 1/2 Earth, so Jupiter is 22x Mars.

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?
Answer: If Saturn is 10 x Venus, and Jupiter is 11 x Earth, Jupiter is 11/10 times Saturn.

Problem 4 - Mercury is 3/4 the size of Mars. How large is Earth compared to Mercury?
Answer: Mars is 1/2 x Earth, so Mercury is 3/4 x 1/2 = 3/8 x Earth

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?
Answer: Neptune was 4x Venus, but since Venus = earth and Neptune=Uranus, we have Uranus = 4x Earth.

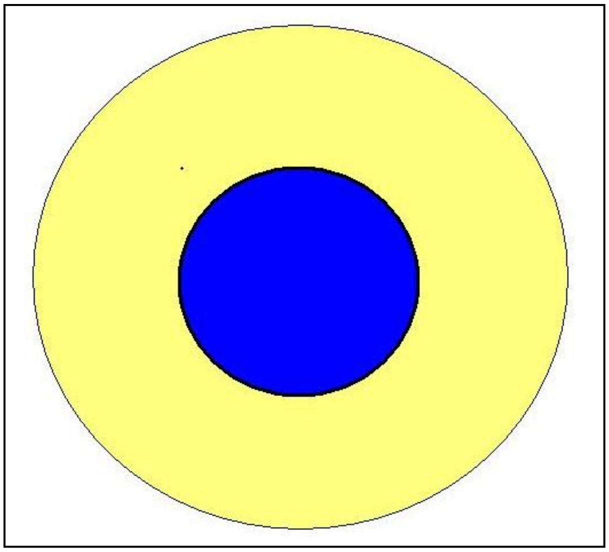
Problem 6 - The satellite of Saturn, called Titan, is 1/10 the size of Uranus. How large is Titan compared to Earth?
Answer: Titan / Uranus = 1/10, but Uranus/Earth = 4, so Titan/Earth = 3/10 x 4 = 2/5.

Problem 7 - The satellite of Jupiter, called Ganymede, is 2/5 the size of Earth. How large is it compared to Jupiter?
Answer: Earth = 1/11 Jupiter so Ganymede is 1/11 x 2/5 = 2/55 x Jupiter.

Problem 8 - The Dwarf Planet Pluto is 1/3 the size of Mars. How large is Jupiter compared to Pluto?
Answer: Jupiter = 1/11 Earth, Mars= 1/2 Earth, so Pluto= 1/3 x 1/2 = 1/6 Earth, and 1/66 Jupiter.

Problem 9 – **Answer: Students should, very nearly, reproduce the numbers in the table at the top of the page.**

How to Build a Planet from the Inside Out!



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a mantle consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the shell volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the mantle of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Problem 1 - The planet is a sphere whose total volume is given by $V = 4/3 \pi R^3$. The total volume is found by adding the volumes of the core and shell to get $V = 4.18 \times 10^{12} + 2.92 \times 10^{13} = 3.34 \times 10^{13}$ cubic kilometers. Then solving the equation for R we get $R = (3.34 \times 10^{13} / (1.33*3.14))^{1/3}$ = 19,978 kilometers. Since the data are only provided to 3 place accuracy, the final answer can only have three significant figures, and with rounding this equals a radius of R = **20,000 kilometers.**

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet?
Answer: V = 4.18×10^{12} cubic kilometers / 1.1×10^{12} cubic kilometers = **4 Earths.**

B) How many Earths could fit inside the mantle of this hypothetical planet?
Answer: V = 2.92×10^{13} cubic kilometers / 1.1×10^{12} cubic kilometers = **27 Earths.**

Problem 3 - What is A) the mass of this planet in kilograms? Answer: $8.3 \times 6.0 \times 10^{24}$ kilograms = **5.0×10^{25} kilograms.**
B) The average density of the planet in kilograms/cubic meter?
Answer: Density = total mass/ total volume
= 5.0×10^{25} kilograms/3.34×10^{13} cubic kilometers
= 1.5×10^{12} kilograms/cubic kilometers.
Since 1 cubic kilometer = 10^9 cubic meters,
= 1.5×10^{12} kilograms/cubic kilometers x (1 cubic km/10^9 cubic meters)
= **1,500 kilograms/cubic meter.**

Problem 4 - We have to subtract the total mass of the ice shell from the mass of the planet to get the mass of the core, then divide this by the volume of the core to get its density. Mass = Density x Volume, so the shell mass is 1,000 kg/ $m^3 \times 2.92 \times 10^{13}$ $km^3 \times (10^9\ m^3/km^3) = 2.9 \times 10^{25}$ kg. Then the core mass = 5.0×10^{25} kilograms - 2.9×10^{25} kg = 2.1×10^{25} kg. The core volume is 4.18×10^{12} $km^3 \times (10^9\ m^3/km^3) = 4.2 \times 10^{21}\ m^3$, so the density is $D = 2.1 \times 10^{25}$ kg / $4.2 \times 10^{21}\ m^3$= **5,000 kg/ m^3.**

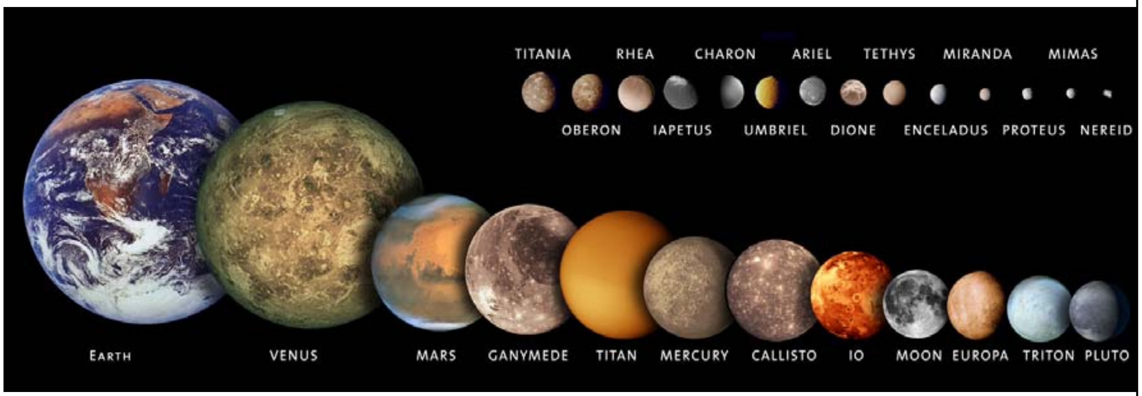
Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?
Answer: **Basalt.**

Note that, although the average density of the planet (1,500 kg/m^3) is not much more than solid ice (1,000 kg/m^3), the planet has a sizable rocky core of higher density material. Once astronomers determine the size and mass of a planet, these kinds of 'shell-core' models can give valuable insight to the composition of the interiors of planets that cannot even be directly imaged and resolved! In more sophisticated models, the interior chemistry is linked to the temperature and location of the planet around its star, and proper account is made for the changes in density inside a planet due to compression and heating. The surface temperature, which can be measured from Earth by measuring the infrared 'heat' from the planet, also tells which compounds such as water can be in liquid form.

Big Moons and Small Planets!



This diagram shows the Top-26 moons and small planets in our solar system, and drawn to the same scale.

Problem 1 – What fraction of the objects are smaller than our moon?

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Problem 4 – If Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer Key

Problem 1 – What fraction of the objects are smaller than our moon?
Answer: **17/26**

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Answer: Io, Callisto, Titan and Ganymede : 4/26 or **2/13**

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Answer: Moon, Europa, Triton and Pluto so 4/26 = **2/13**.

Problem 4 – Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Answer: ½ x 1/6 = **1/12 the size of Earth**.

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer: Oberon is 1/7 the diameter of Earth.
Titania is 4/9 the diameter of Io, and Io is 1/3 the diameter of Earth
So Titania is (4/9) x (1/3) the diameter of Earth
So Titania is 4/27 the diameter of Earth.

Comparing Oberon, which is 1/7 the diameter of Earth with Titania, which is 4/27 the diameter of Earth, which fraction is larger: 1/7 or 4/27?

Find the common denominator 7 x 27 = 189, then cross-multiply the fractions:

Oberon: 1/7 = 27/189 and Titania: 4/27 = (4x7)/189 = 28/189 so

Titania is 28/189 Earth's diameter and Oberon is 27/189 Earth's diameter, and so **Titania is slightly larger!**

Finding Mass in the Cosmos

$$Fg = \frac{G M m}{R^2}$$

$$Fc = \frac{m V^2}{R}$$

$$V = \frac{2 \pi R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4 \pi^2 R^3 / (G T^2)}$ where $G = 6.6726 \times 10^{-11}$ $N\text{-}m^2/kg^2$ and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below. (Note: Make sure all units are in meters and seconds first! 1 light years = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{G M m}{R^2} = \frac{m V^2}{R}$$

Cancil the companion mass, m, on both sides, and isolate the primary mass, M, on the left side:

$$M = \frac{R V^2}{G}$$

Now use the definition of V to eliminate it from the equation.

$$M = \frac{R}{G}\left(\frac{2 \pi R}{T}\right)^2$$

and simplify

$$M = \frac{4 \pi^2 R^3}{G T^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1×10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1×10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9×10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3×10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4×10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9×10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1×10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6×10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2×10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7×10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.

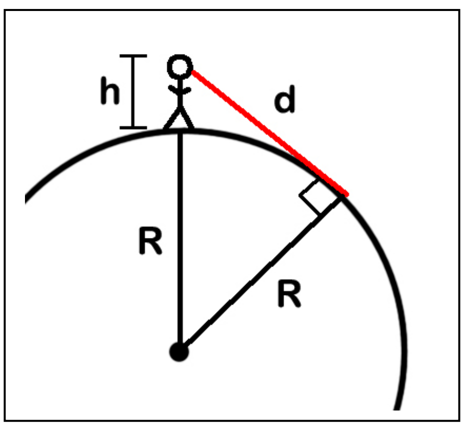
The Distance to the Martian Horizon

Imagine you and your friend standing on the surface of a perfectly flat planet. Your friend starts walking away from you, and you see her size get smaller and smaller until at a distance of 100 kilometers you can't even see her at all.

Now imagine the same experiment on a spherical planet. As many sea-farers discovered 1000 years ago, because Earth is curved, you will see the ships hull disappear from the bottom upwards, then the last thing that vanishes is the top of the main mast.

The image above comes from Johannes de Sacrobosco's Tractatus de Sphaera (*On the Sphere of the World*) written in 1230 AD. It showcases the knowledge that the appearance of ships on the horizon testified to a curved earth. A bit of simple geometry, and some help from the Pythagorean Theorem, will let you calculate the distance to the horizon on Mars as viewed from the InSight Lander!

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.



Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.

Answer: $d^2 = (R+h)^2 – R^2$

So $d^2 = 2Rh + h^2$

And so $d = (2Rh + h^2)^{1/2}$

Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Answer: For the formula to work, R and h must be in the same units of meters (or kilometers!). When h is much less then R, the quantity h^2 is always much, much smaller than 2Rh, so the formula simplifies to $d = (2Rh)^{1/2}$.

For Mars, h = 1 meter and R = 3378000 meters and so **d = 2599 meters or about 2.6 kilometers.**

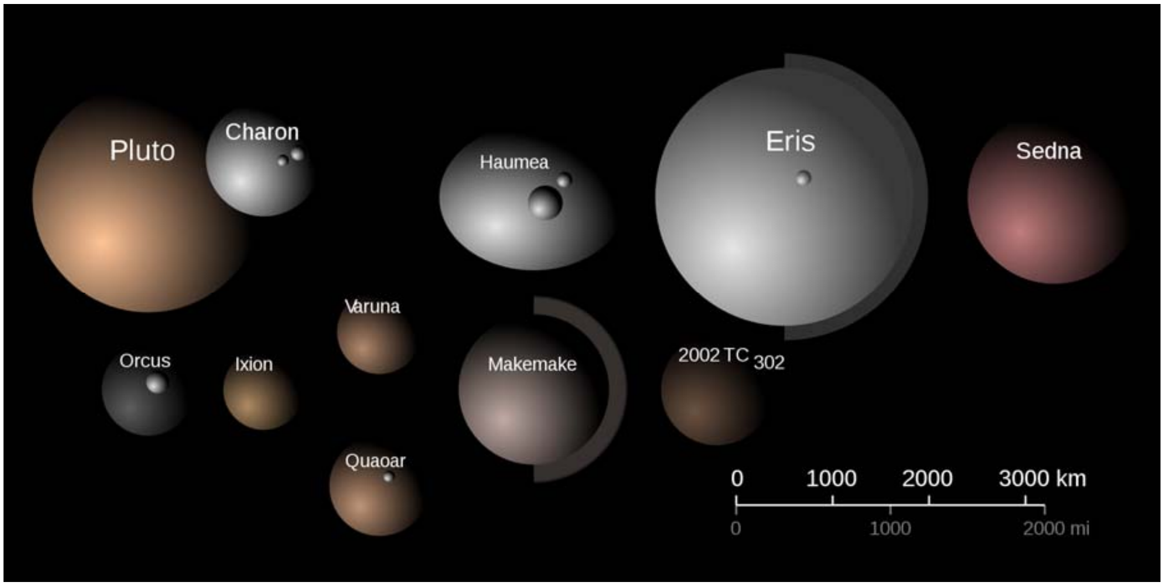
Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Answer: h = 100 meters or 0.1 km, so the horizon distance for one cell tower has a radius of $d = (2 \times 3378 \text{ km} \times 0.1 \text{ km})^{1/2}$ = 26 kilometers. The area of this reception circle is $A = \pi R^2 = 2123 \text{ km}^2$. The total surface area of Mars is $A = 4 \times \pi \times (3378)^2 = 1.43\times10^8 \text{ km}^2$.

Then dividing the surface area of Mars by the cell tower reception area we get $N = 1.43\times10^8/2123$ = **67,530 cell towers**.

As of 2013, there are over 200,000 cell towers in the Unites States alone!

Dwarf Planets and Kepler's Third Law



Object	Distance (AU)	Period (years)
Mercury	0.4	0.24
Venus	0.7	0.61
Earth	1.0	1.0
Mars	1.5	1.88
Ceres	2.8	4.6
Jupiter	5.2	11.9
Saturn	9.5	29.5
Uranus	19.2	84.0
Neptune	30.1	164.8
Pluto	39.4	247.7
Ixion	39.7	
Huya	39.8	
Varuna	42.9	
Haumea	43.3	285
Quaoar	43.6	
Makemake	45.8	310
Eris	67.7	557
1996-TL66	82.9	
Sedna	486.0	

Note: Distances are given in Astronomical Units (AU) where 1 AU = earth-sun distance of 150 million km.

Astronomers have detected over 500 bodies orbiting the sun well beyond the orbit of Neptune. Among these 'Trans-Neptunian Objects (TNOs) are a growing number that rival Pluto in size. This caused astronomers to rethink how they should define the term 'planet'.

In 2006 Pluto was demoted from a planet to a dwarf planet, joining the large asteroid Ceres in that new group. Several other TNOs also joined that group, which now includes five bodies shown highlighted in the table. A number of other large objects, called Plutoids, are also listed.

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit: A) Polynomial function? B) Power-law function?

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers?

Problem 3 - Using your best-fit model, what would you predict for the periods of the TNOs in the table?

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit:

A) Polynomial function? **The N=3 polynomial $D(x) = -0.0005x^3 + 0.1239x^2 + 2.24x - 1.7$**

B) Power-law function? **The N=1.5 powerlaw: $D(x) = 1.0x^{1.5}$**

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers? The two predictions are shown in the table:

Object	Distance	Period	N=3	N=1.5
Mercury	0.4	0.24	-0.79	0.25
Venus	0.7	0.61	-0.08	0.59
Earth	1	1	0.66	1.00
Mars	1.5	1.88	1.93	1.84
Ceres	2.8	4.6	5.53	4.69
Jupiter	5.2	11.9	13.22	11.86
Saturn	9.5	29.5	30.33	29.28
Uranus	19.2	84	83.44	84.13
Neptune	30.1	164.8	164.34	165.14
Pluto	39.4	247.7	248.31	247.31
Ixion	39.7		251.21	250.14
Huya	39.8		252.19	251.09
Varuna	42.9		282.94	280.99
Haumea	43.3	285	286.99	284.93
Quaoar	43.6		290.05	287.89
Makemake	45.8	310	312.75	309.95
Eris	67.7	557	562.67	557.04
1996-TL66	82.9		750.62	754.80
Sedna	486		-27044.01	10714.07

Answer: The N=3 polynomial gives negative periods for Mercury, Venus and Sedna, and poor answers for Earth, Mars, Ceres and Jupiter compared to the N=3/2 power-law fit. The N=3/2 power-law fit is the result of Kepler's Third Law for planetary motion which states that the cube of the distance is proportional to the square of the period so that when all periods and distances are scaled to Earth's orbit, Period = Distance$^{3/2}$

Problem 3 - See the table above for shaded entries