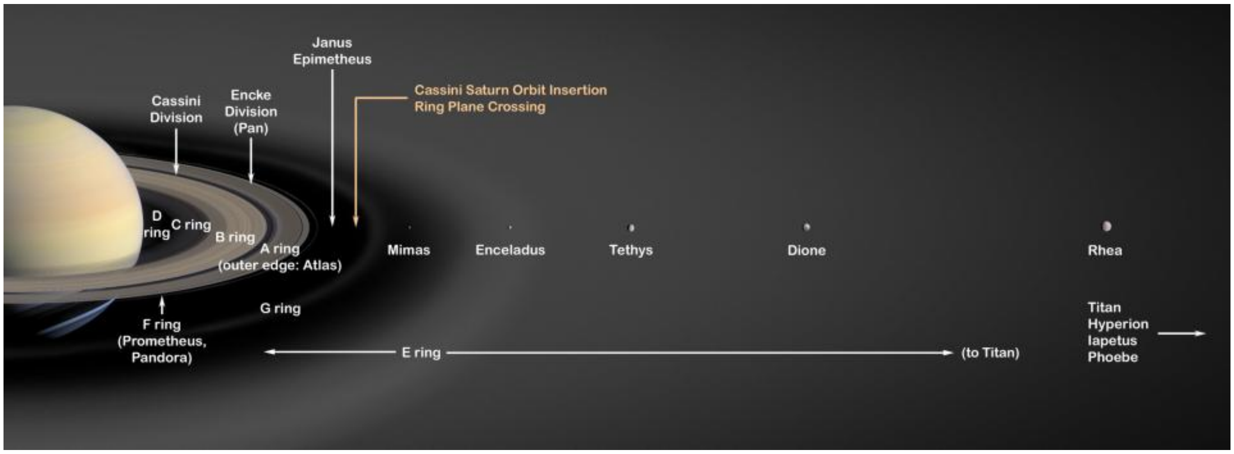
The Rings of Saturn



Saturn, the most beautiful planet in our solar system, is famous for its dazzling rings. Shown in the figure above, these rings extend far into space and engulf many of Saturn's moons. The brightest rings, visible from Earth in a small telescope, include the D, C and B rings, Cassini's Division, and the A ring. Just outside the A ring is the narrow F ring, shepherded by tiny moons, Pandora and Prometheus. Beyond that are two much fainter rings named G and E. Saturn's diffuse E ring is the largest planetary ring in our solar system, extending from Mimas' orbit to Titan's orbit, about 1 million kilometers (621,370 miles).

The particles in Saturn's rings are composed primarily of water ice and range in size from microns to tens of meters. The rings show a tremendous amount of structure on all scales. Some of this structure is related to gravitational interactions with Saturn's many moons, but much of it remains unexplained. One moonlet, Pan, actually orbits inside the A ring in a 330-kilometer-wide (200-mile) gap called the Encke Gap. The main rings (A, B and C) are less than 100 meters (300 feet) thick in most places. The main rings are much younger than the age of the solar system, perhaps only a few hundred million years old. They may have formed from the breakup of one of Saturn's moons or from a comet or meteor that was torn apart by Saturn's gravity.

Problem 1 – The dense main rings extend from 7,000 km to 80,000 km above Saturn's equator (Saturn's equatorial radius is 60,300 km). If the average thickness of these rings is 1 kilometer, what is the volume of the ring system in cubic kilometers? (use $\pi = 3.14$)

Problem 2 – The total number of ring particles is estimated to be 3×10^{16} . If these ring particles are evenly distributed in the ring volume calculated in Problem 1, what is the average distance in meters between these ring particles?

Problem 3 – If the ring particles are about 1 meter in diameter and have the density of water ice, 1000 kg/m^3, about what is the diameter of the assembled body from all of these ring particles?

Problem 1 – The dense main rings extend from 7,000 km to 80,000 km above Saturn's equator (Saturn's equatorial radius is 60,300 km). If the average thickness of these rings is 1 kilometer, what is the volume of the ring system in cubic kilometers? (use $\pi = 3.14$)

Answer: The area of a ring with an inner radius of r and an outer radius of R is given by $A = \pi (R^2 - r^2)$ and its volume for a thickness of h is just $V = \pi (R^2 - r^2)h$.

For Saturn's rings we have an inner radius r = 60300km+7000km =67300 km and an outer radius of R = 60300km+80000km = 140,300 km, and a volume of $V = 3.14 ((140300)^2 - (67300)^2) (1.0)$ = **4.75×10^{10} km^3**.

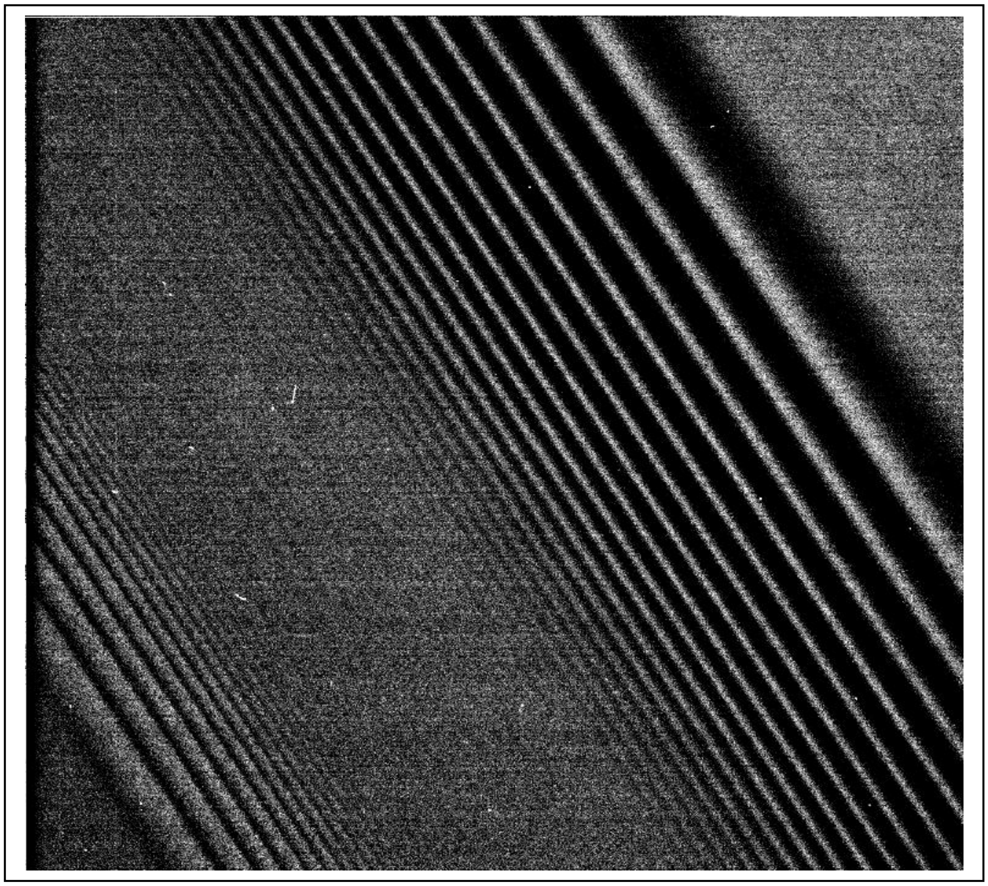
Problem 2 – The total number of ring particles is estimated to be 3×10^{16}. If these ring particles are evenly distributed in the ring volume calculated in Problem 1, what is the average distance in meters between these ring particles?

Answer: 4.75×10^{10} km^3 / 3×10^{16} = 1.6×10^{-6} km^3/particle, so each particle is found in a volume of 1.6×10^{-6} km^3. For two cubes next to each other each with a volume of $V = s^3$, their centers are separated by exactly s. The distance to the nearest ring particle is $D = (1.6\times10^{-6}\text{ km}^3)^{1/3}$ = 0.012 km, or **12 meters**!

Problem 3 – If the ring particles are about 1 meter in diameter and have the density of water ice, 1000 kg/m^3, about what is the diameter of the assembled body from all of these ring particles?

Answer: The volume of a single particle is $4/3\pi(1/2)^3 = 0.5$ meter^3. The total volume of all the 3×10^{16} particles is then $V = 1.5\times10^{16}$ meter^3. If this is a spherical body then $4/3\pi R^3 = 1.5\times10^{16}$ m^3, so R = 151185 meters or 150 kilometers in radius. The diameter is then **300 kilometers**.

Saturn's Rings – A close-up study



This spectacular close-up image of Saturn's A ring was taken in 2004 by the Cassini spacecraft. It shows a 220-km wide snapshot of a magnified portion of the A ring, and how it dissolves into smaller ringlets. Astronomers think that these ringlets are formed by gravitational interactions with Saturn's inner moons, causing ripples and waves to form that 'bunch up' billions of ring particles into separate ringlets. Some of the bright spots you see in the dark bands may be 'shepherding moonlets' only a few kilometers in size, which keep the ring particles orbiting together.

Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers.

Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer?

Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?

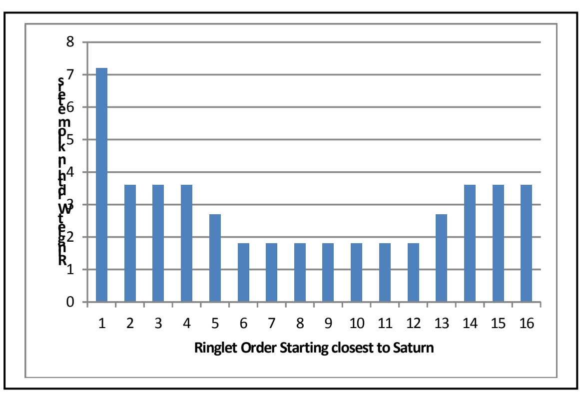
Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image. Answer: When printed on standard 8 ½ x 11 paper, the width of the image is 124 millimeters, so the scale is 220 km/124mm = **1.8 km/mm**.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers. Answer: **See below**.

Ringlet	millimeters	kilometers
1	4	7.2
2	2	3.6
3	2	3.6
4	2	3.6
5	1.5	2.7
6	1	1.8
7	1	1.8
8	1	1.8
9	1	1.8
10	1	1.8
11	1	1.8
12	1	1.8
13	1.5	2.7
14	2	3.6
15	2	3.6
16	2	3.6

Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer? Answer: (6x3.6 + 7.2 +2x2.7+7x1.8)/16 = **2.9 kilometers**.

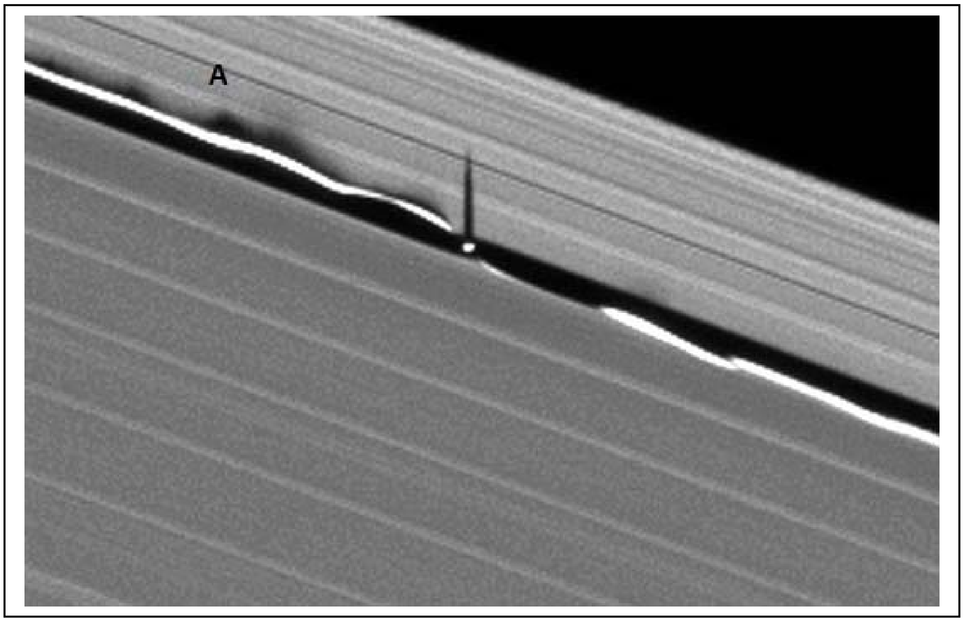
Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?



Answer: The ringlet widths decrease as you get further from Saturn and are present with distinct dark gaps in between. As the dark gaps become narrower than about 3 km near Ringlet 12, the rings begin to increase slightly in size.

It is ‘interesting’ that the ringlets in the lower right corner are wider than the upper ringlets, and there are fewer of them.

Saturn's Rings – Shadows from Moons and Ringlets



Never-before-seen looming vertical structures, created by the tiny moon Daphnis, cast long shadows across Saturn's A Ring in this startling image taken by the Cassin spacecraft. The 8-kilometre-wide moon Daphnis orbits within the 42-kilometre-wide Keeler Gap in Saturn's outer A Ring, and its gravitational pull perturbs the orbits of the particles forming the gap's edges. The Keeler Gap is foreshortened and appears only about 30 km wide because the image was taken at an angle of about 45 degrees to the ring plane.

Problem 1 – If the apparent perpendicular width of the Keeler Gap is 30 km, what is the length of the shadow of Daphnis in this image?

Problem 2 – Rounded to the nearest kilometer, what is the length of Shadow A to the left from Daphnis?

Problem 3 – Create a scaled model of this ring area and its 45 degree inclination, and using right triangles, estimate the elevation angle of the sun above the ring plane.

Problem 4 – From your scaled model, what is the height of the feature that is casting Shadow A on the ring plane?

Problem 1 – If the apparent (foreshortened) perpendicular width of the Keeler Gap is 30 km, what is the corresponding foreshortened length of the shadow of Daphnis in this image?

Answer: After printing this page on standard 8 ½ x 11 paper, the perpendicular width of the Keeler Gap is about 4 millimeters, which corresponds to 30 km, so the scale is 30 km/4mm = 7.5 km/mm. The length of the moonlets shadow is 12 mm, so its projected length is 12 x 7.5 = **90 km**.

Problem 2 – Rounded to the nearest kilometer, what is the length of Shadow A to the left from Daphnis?

Answer: Students may obtain 5 mm if they measure from the inner edge of the white band, or 3 mm if they measure from the outer edge of the white band. These measurements correspond to **23 km or 38 km**.

Problem 3 – Create a scaled model of this ring area and its 45 degree inclination, and using right triangles, estimate the elevation angle of the sun above the ring plane.

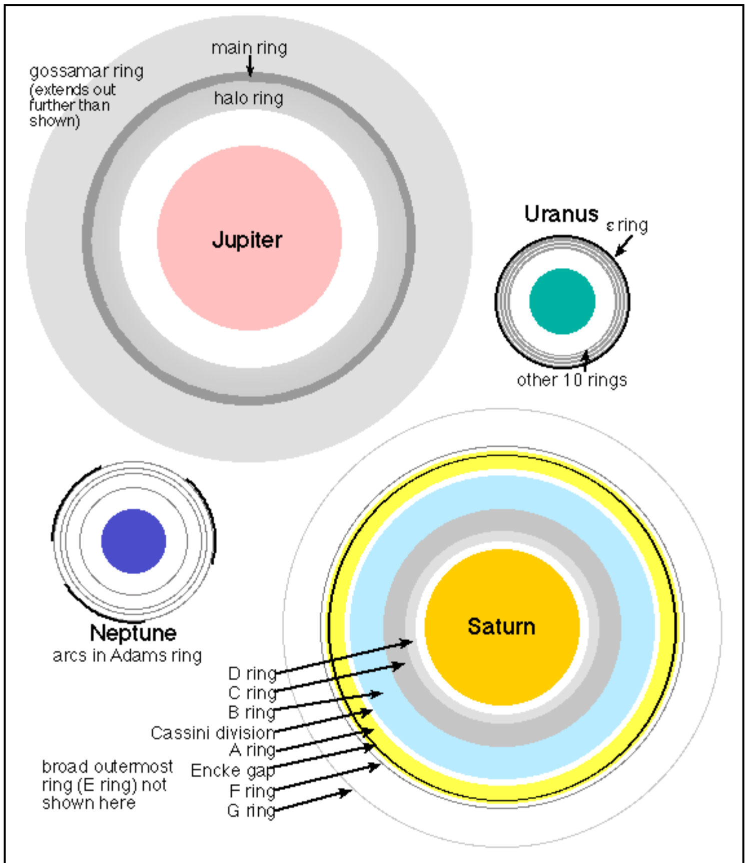
Answer: For the shadow of Daphnis, the true shadow length is the hypotenuse of a 45-45-90 right triangle ABD, and its shadow length in the image is the horizontal side along segment BC of this triangle, so the hyponenuse (segment AB) length is 90 km x $(2)^{\mathbf{1/2}}$ = 127 km. The diameter of the moon is 8 km, so the sun angle for the shadow on the ring plane is a triangle whose sides are AC=8 km and BC = 127 km. The angle to the sun ABC can be measured with a protractor and is Tan(Theta) = 8km/127km so to the nearest degree, **Theta = 4 degrees**.

Problem 4 – From your scaled model, what is the height of the feature that is casting Shadow A on the ring plane?

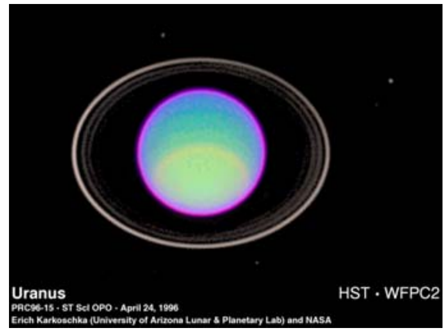
Answer: The projected height of the shadow is 23 or 38 km. The true shadow length is then
23 x $(2)^{\mathbf{1/2}}$ = 33 km or
38 x $(2)^{1/2}$ = 54 km.

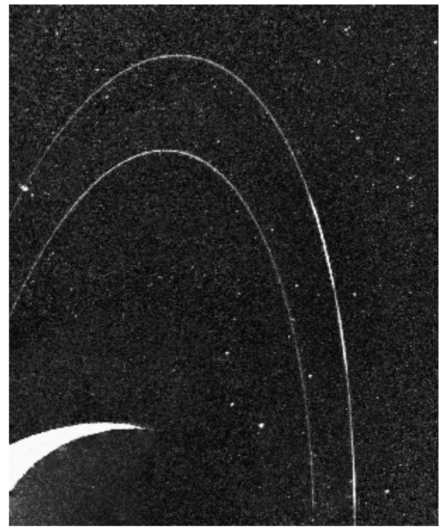
The sun angle above the ring plane is 4 degrees, so if the right triangle ABC has side BC = 33 km then the height of segment AC can be measured with a protractor as about 33xTan(4) = **2.3 km**, or using the second shadow measurement 54xtan(4) = **3.8 km**.

Note: Astronomers find it amazing that even though the A ring has a thickness of about 10 meters, the wave produced by this small moon rises over 200 times higher above the ring plane than the thickness of the rings themselves!



Rings of the jovian planets shown to the same scale
Courtesy of Nick Strobel at www.astronomynotes.com





The rings of Uranus (top right) and Neptune (bottom right) are similar to those of Jupiter and consist millions of small rocky and icy objects in separate orbits. The figure shows a comparison of the scales of the planets and their ring systems.

Problem 1 – Compare the extent of the ring systems for each planet in terms of their size in units of the radius of the corresponding planet. For example, 'The rings of Uranus extend from 1.7 to 2.0 times the radius of Uranus'.

Problem 2 – An icy body will be destroyed by a planet if it comes within the Tidal Limit of the planet. At this distance, the difference in gravity between the near and the far side of the body exceeds the body's ability to hold together by its own gravity, and so it is shredded into smaller pieces. For Jupiter (2.7), Saturn (2.2), Uranus (2.7) and Neptune (2.9), the Tidal Limits are located between 2.2 and 2.9 times the radius of each planet from the planet's center. Describe where the ring systems are located around each planet compared to the planets Tidal Limit. Could the rings be explained by a moon or moon's getting too close to the planet?

Problem 1 – Compare the extent of the ring systems for each planet in terms of their size in units of the radius of the corresponding planet. For example, 'The rings of Uranus extend from 1.7 to 2.0 times the radius of Uranus'.

Answer:

Jupiter	from	1.4 to 2.3
Saturn	from	1.1 to 3.6
Uranus	from	1.7 to 2.0
Neptune	from	1.7 to 2.7

Problem 2 – An icy body will be destroyed by a planet if it comes within the Tidal Limit of the planet. At this distance, the difference in gravity between the near and the far side of the body exceeds the body's ability to hold together by its own gravity, and so it is shredded into smaller pieces. For Jupiter (2.7), Saturn (2.2), Uranus (2.7) and Neptune (2.9), the Tidal Limits are located between 2.2 and 2.9 times the radius of each planet from the planet's center. Describe where the ring systems are located around each planet compared to the planets Tidal Limit. Could the rings be explained by a moon or moon's getting too close to the planet?

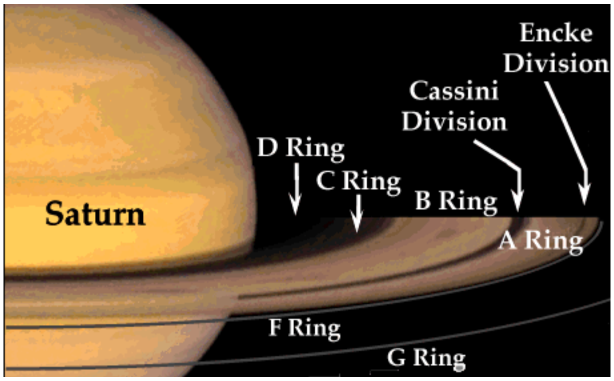
Answer:

Jupiter	1.4		2.3	(2.7)			
Saturn	1.1	(2.2)					3.6
Uranus	1.7	2.0		(2.7)			
Neptune	1.7				2.7	(2.9)	

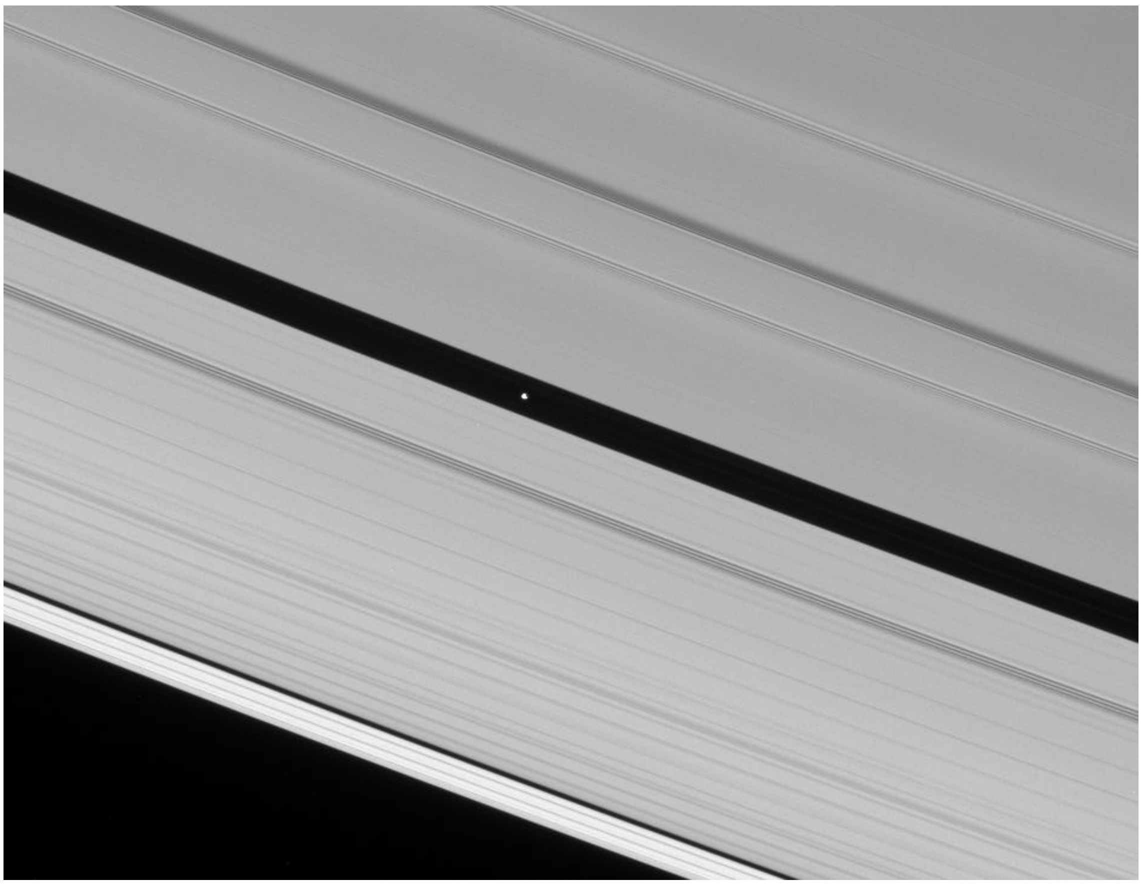
The location of the tidal radius for each planet is given in parenthesis on this scaled model. We see that for all of the planets, most of the ring material is located inside the Tidal Limit. In the case of Saturn, which seems to be the exception, there is also some ring material outside R = 2.2 Rs and includes the very sparse F and G rings. But even for Saturn, the majority of the visible rings (seen through a telescope) are inside the Tidal Limit.

The rings can be explained by moons that were tidally destroyed as they passed inside the Tidal Limit for each planet. These moons could not have been formed this close because the same tidal forces that destroy these moons would have prevented them from assembling, so the moons must have been formed outside the Tidal Limit and over millions of years their orbits carried them closer and closer to the Tidal Limit until they were finally destroyed.

Pan's Highway - Saturn's Rings



The Encke Gap is a prominent feature of Saturn's outer A-ring system that has been observed since the 1830's. The arrival of the Cassini spacecraft in July 2004 revealed the cause for this gap. A small moonlet called Pan clears out the ring debris in this region every 12 hours as it orbits Saturn!

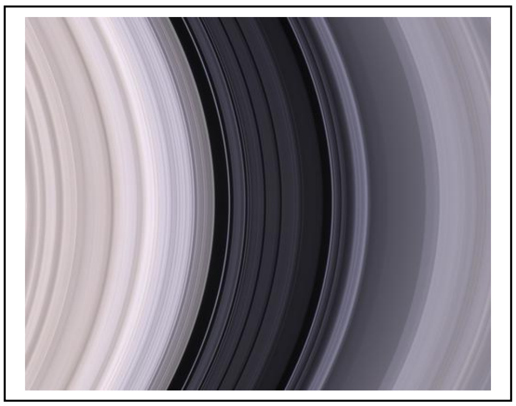


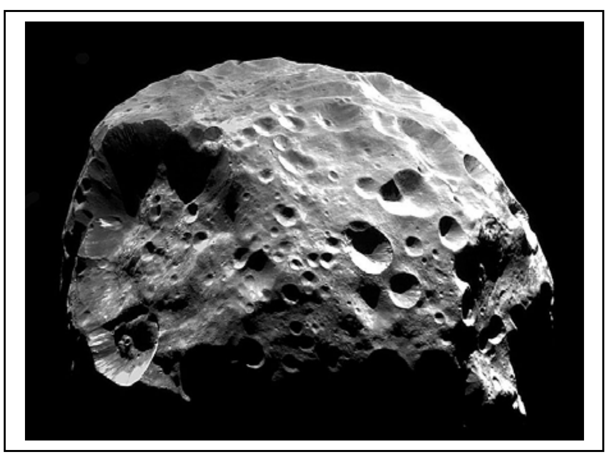
Problem 1 - This image was taken by Cassini in 2007 and at the satellite's distance of 1 million kilometers, spans a field of view of 5,700 km x 4,400 km. With the help of a millimeter ruler, what is the scale of the image in kilometers per millimeter?

Problem 2 - Pan is that bright spot within the black zone of the Encke Gap. About how many kilometers in diameter is Pan?

Problem 3 - About how wide is the Encke Gap?

Problem 4 - About what is the smallest feature you can discern in the photo?





Problem 1 - The width of the picture is 150 millimeters, so the scale is 5,700 km/150 mm = **38 km/mm.**

Problem 2 - Pan is about 1.0 millimeters in diameter which is 38 km/mm x 1mm = **38 kilometers in diameter.**

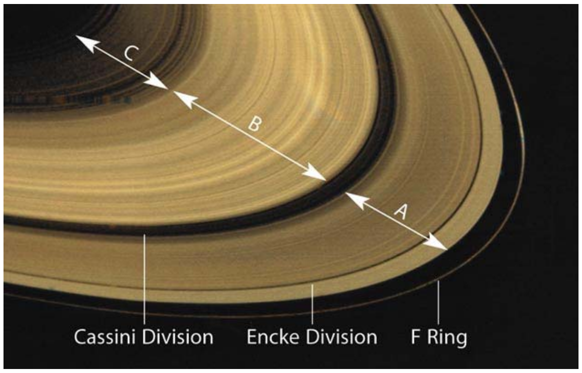
Problem 3 - Students should measure a width of about 5.0 millimeters which is 38 km/mm x 5.0 mm = **190 kilometers**. The actual width of the Encke Gap is 325 km, but projection effects will foreshorten the gap as it appears in the photo. With the actual gap width (325 km) as the hypotenuse, and 190 km as the short side, the angle opposite the short side is the viewing angle of the camera relative to the ring plane. This angle can be found by constructing a scaled triangle and using a protractor to measure the angle, which will be about 36 degrees.

Problem 4 -

It is difficult to estimate lengths smaller than a millimeter. Students may consider using a photocopying machine to make a more convenient enlargement of the image, then measure the features more accurately. Small dark ring bands are about 0.1 mm wide, which is **about 4 km.**

NASA/Cassini mages, top to bottom:
Saturn Rings closeup showing Cassini Division and Encke Gap;
Rings closeup showing detail;
One of Saturn's outer satellites, Phoebe, is about 200 km across, and may have been a captured comet.

Orbit Speeds and Times for Saturn's Rings



The trillions of particles in Saturn's rings orbit the planet like individual satellites. Although the rings look like they are frozen in time, in fact, the rings orbit the planet at thousands of kilometers per hour! The speed of each ring particle is given by the formula:

$$V = \frac{29.4}{\sqrt{R}} \text{ km/s}$$

where R is the distance from the center of Saturn to the ring in multiples of the radius of Saturn (R = 1 corresponds to a distance of 60,300 km).

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Answer: R = (60300 km + 7,000 km) / 60300 km = 1.12, so V = 23.6 km/sec
R = 140300 km/60300 km = 2.33, so V = 16.3 km/sec.

Note: The International Space Station orbits Earth at a speed of 7.7 km/s.

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Answer: R = 117580/60300 = 1.95 V = 17.83 km/s
R = 122170/60300 = 2.02 V = 17.52 km/s
The outer edge particles travel about 17.83 – 17.52 = 0.31 km/sec slower than the inner edge particles.

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Answer: Orbit circumference = 2 π r km, but r = 60300 R so C = 2 (3.141) x 60300 R , C = 379,000 R km, wnhere R is in Saturn radius units. Since the orbit speed is V = $24.9/R^{1/2}$, then Time = C/V = 15220 $R^{3/2}$ seconds. Since 1 hour = 3600 seconds, we have **T = 4.22 $R^{3/2}$ hours**.

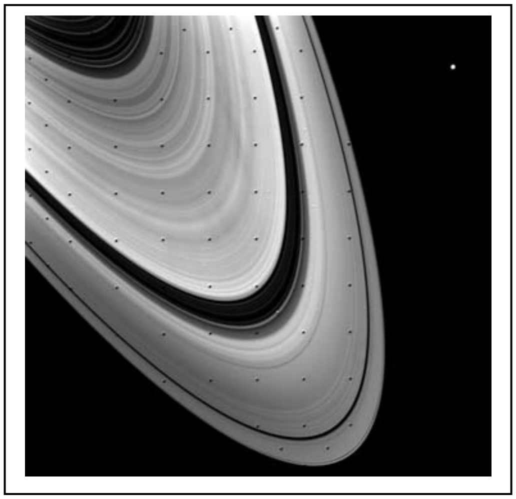
Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Answer: R = 1.95 so T = 11.49 hours.
R = 2.02 so T = 12.11 hours.

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Answer: 22.5/11.49 = 1.94 which is nearly 2.0. This means that every time Mimas orbits once, the particles in the Cassini Division orbit about twice around Saturn. This is an example of an orbit resonance. Because the ring particles encounter a push from Mimas's gravitational field at the same location every two orbits, they will be ejected. This is an explanation for why the Cassini Division has so few ring particles.

How Saturn's Moon Mimas created the Cassini Division



The Cassini Division is easily seen from Earth with a small telescope, and splits the rings of Saturn into two major groups. A little detective work shows that there may be a good reason for this gap that involves Saturn's nearby moon, Mimas.

Mimas orbits Saturn once every 22 hours, and would-be particles in the Cassini Division would orbit once every 11-12 hours, so that the ratio of the orbit periods is close to 2 to 1. This creates a resonance condition where the gravity of Mimas perturbs the Cassini particles and eventually ejects them.

Imagine a pendulum swinging. If you lightly tap the pendulum when it reaches the top of its swing, and do this every other swing, eventually the small taps add up to increasing the height of the pendulum.

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division from Mimas is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67 \times 10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67 \times 10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Answer: Acceleration = 6.67×10^{-11} (4.0×10^{19}) / $(6.7 \times 10^{7}$ meters$)^2$
= **5.9×10^{-7} meters/sec^2**

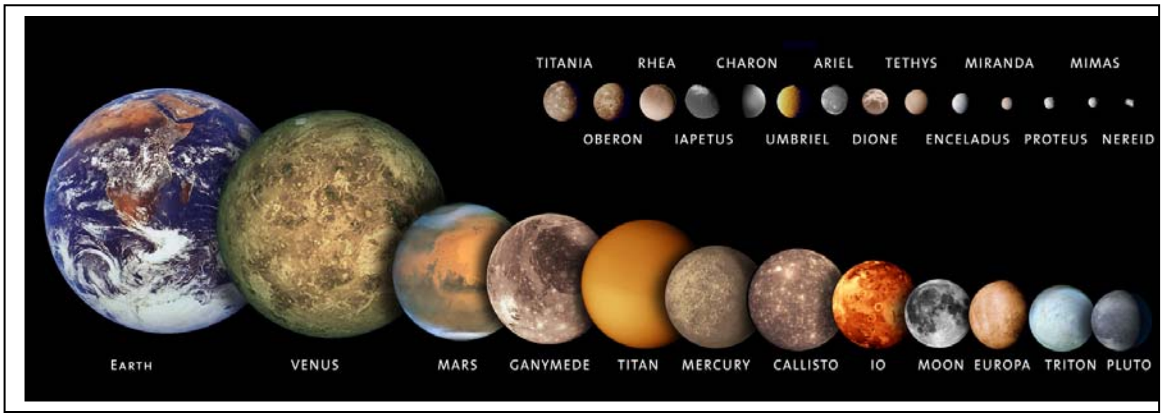
Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Answer: 1 hour = 3600 seconds, so 2 hours = 7200 seconds and
speed = 5.9×10^{-7} m/sec^2 x 7200 sec
= 4.2×10^{-3} meters/sec per orbit.

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Answer: (1000 m/s)/(0.0042 m/s) = 238000 orbits. Since 1 orbit = 12 hours, we have 12 x 238,000 = 2,856,000 hours or about **326 years**.

Big Moons and Small Planets!



This diagram shows the Top-26 moons and small planets in our solar system, and drawn to the same scale.

Problem 1 – What fraction of the objects are smaller than our moon?

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Problem 4 – If Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer Key

Problem 1 – What fraction of the objects are smaller than our moon?
Answer: **17/26**

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Answer: Io, Callisto, Titan and Ganymede : 4/26 or **2/13**

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Answer: Moon, Europa, Triton and Pluto so 4/26 = **2/13**.

Problem 4 – Saturn's moon Titan is ½ the diameter of Earth, and Saturn's moon Dione is 1/6 the diameter of Titan, how large is the diameter of Dione compared to Earth?

Answer: ½ x 1/6 = **1/12 the size of Earth**.

Problem 5 – Oberon is 1/7 the diameter of Earth, Io is 1/3 the diameter of Earth, and Titania is 4/9 the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer: Oberon is 1/7 the diameter of Earth.
Titania is 4/9 the diameter of Io, and Io is 1/3 the diameter of Earth
So Titania is (4/9) x (1/3) the diameter of Earth
So Titania is 4/27 the diameter of Earth.

Comparing Oberon, which is 1/7 the diameter of Earth with Titania, which is 4/27 the diameter of Earth, which fraction is larger: 1/7 or 4/27?

Find the common denominator 7 x 27 = 189, then cross-multiply the fractions:

Oberon: 1/7 = 27/189 and Titania: 4/27 = (4x7)/189 = 28/189 so

Titania is 28/189 Earth's diameter and Oberon is 27/189 Earth's diameter, and so **Titania is slightly larger!**