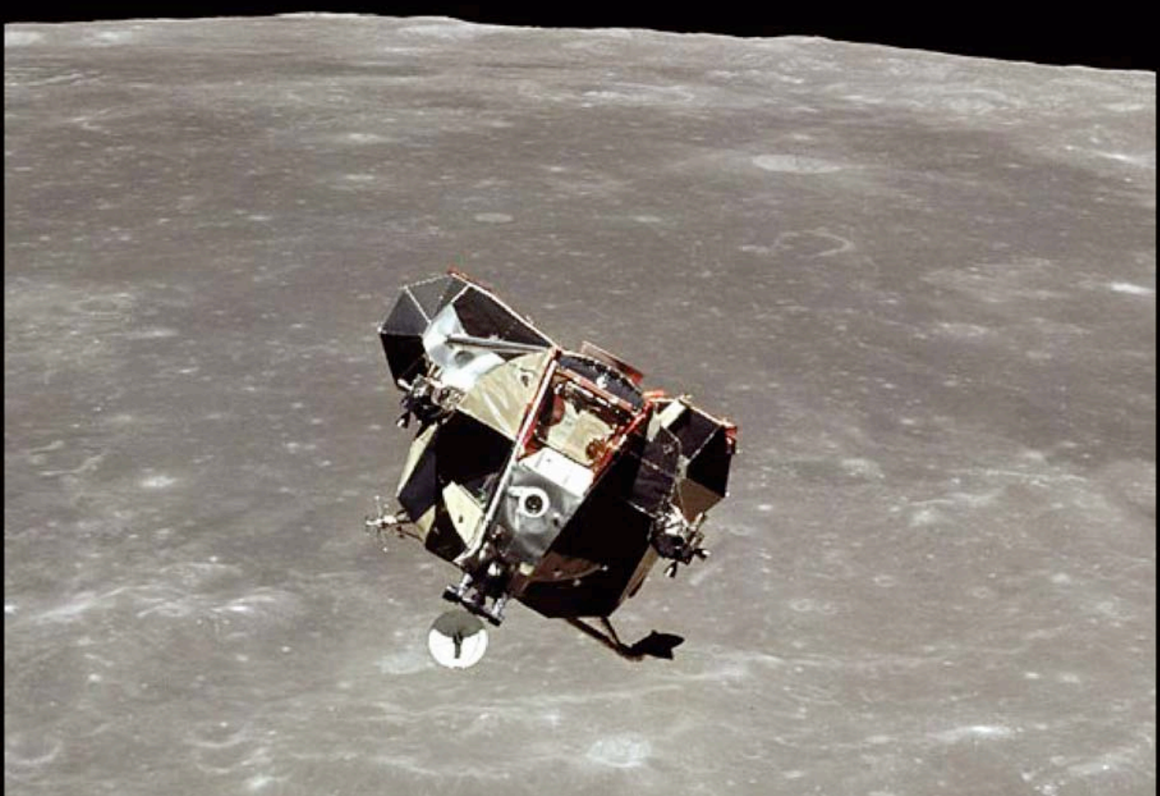
National Aeronautics and Space Administration

Lunar Math



This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2005-2012 school years. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide, and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

<u>Front and back cover credits</u>: **Front: Colony on moon (NASA); Solar eclipse (Fred Espenak); Apollo-11 LEM and earthrise (NASA). Back: The Lunar Rover**

This booklet was created through an education grant NNH06ZDA001N-EPO from NASA's Science Mission Directorate.

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Mathematics Topic Matrix

Topic	Problem Numbers																														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers				X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			X	X		
Numbers, patterns, percentages		X	X																					X	X	X	X	X	X		
Averages																														X	X
Time, distance,speed																															
Mass, density, volume																									X						
Areas and volumes																		X		X					X		X	X	X		
Scale drawings	X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			X	X		
Geometry																			X		X										X
Scientific Notation																											X				
Unit Conversions				X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			X	X		
Fractions	X																									X					
Graph or Table Analysis		X	X																											X	
Solving for X																															
Evaluating Fns																															
Modeling																															
Probability																										X	X				
Rates/Slopes																															
Logarithmic Fns																															
Polynomials																															
Power Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																			X												
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																								
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
Inquiry																									
Technology, rulers	X	X	X	X		X	X					X													
Numbers, patterns, percentages					X	X															X				
Averages																									
Time, distance,speed					X		X	X																	
Mass, density, volume																				X		X			
Areas and volumes		X										X					X	X	X	X					
Scale drawings	X	X	X	X		X	X		X	X		X		X										X	
Geometry					X				X	X		X	X	X										X	
Scientific Notation										X								X	X	X		X	X	X	X
Unit Conversions	X	X	X	X		X	X					X	X				X					X	X		
Fractions																									
Graph or Table Analysis							X	X			X				X										
Solving for X											X				X										
Evaluating Fns										X						X			X		X	X	X		
Modeling										X	X										X		X		
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How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Mathematics offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for Lunar data.

Lunar Math is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Lunar Math**. Read the scenario that follows

Mr. Smith and Ms. Green decided to team teach. Mr. Smith is teaching students about the solar system and the history of the Apollo Missions, looking toward the future of the next landings on the moon, in 2017. Ms. Green is teaching several math levels, with the same students that Mr. Smith teaches science. They decide to use the Lunar Math supplement to learn about the science and how the math applications will provide information about the moon. Ms. Green checks the *Alignment to Mathematical Standards*, in the front of Lunar Math, to determine which topics she can use with her 5 classes. All classes can use a review on scale drawings, probability and beginning geometry found in activities 1-5. This is where they begin. Mr. Smith has the students’ look at images on the internet of the moon, they begin with images provided by the Lunar and Planetary Institute and build a knowledge base about the surface of the moon. Ms. Green uses the first 5 activities from Lunar Math as students learn about the size of craters that impact the moon’s unprotected surface.

Lunar Math can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and space discovery.

Alignment with Standards

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Teacher Comments

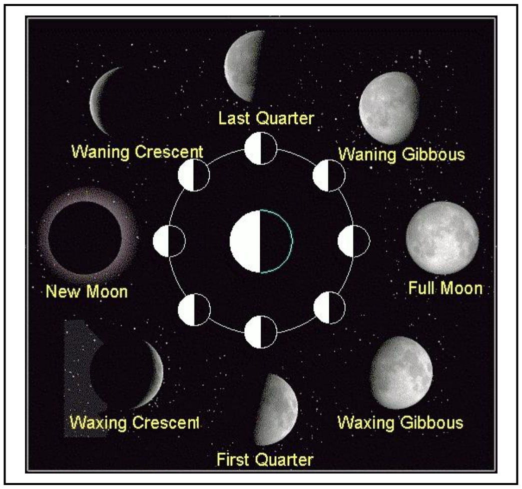
"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

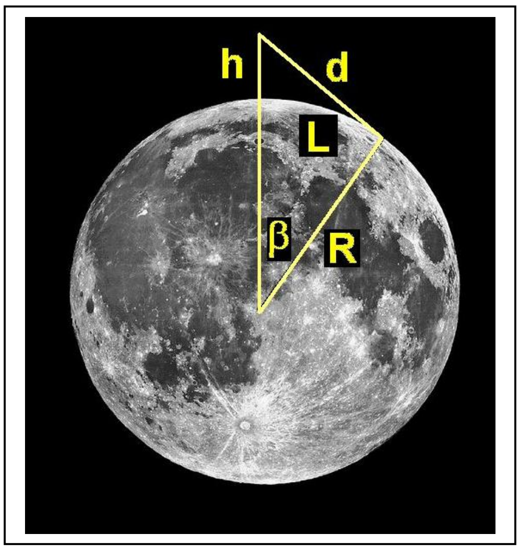
"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

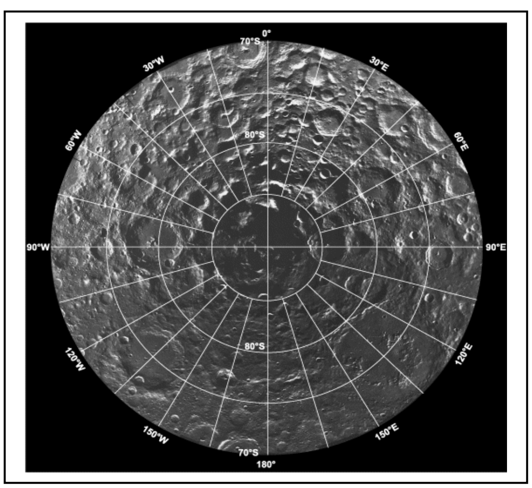
"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

Introducing the Moon







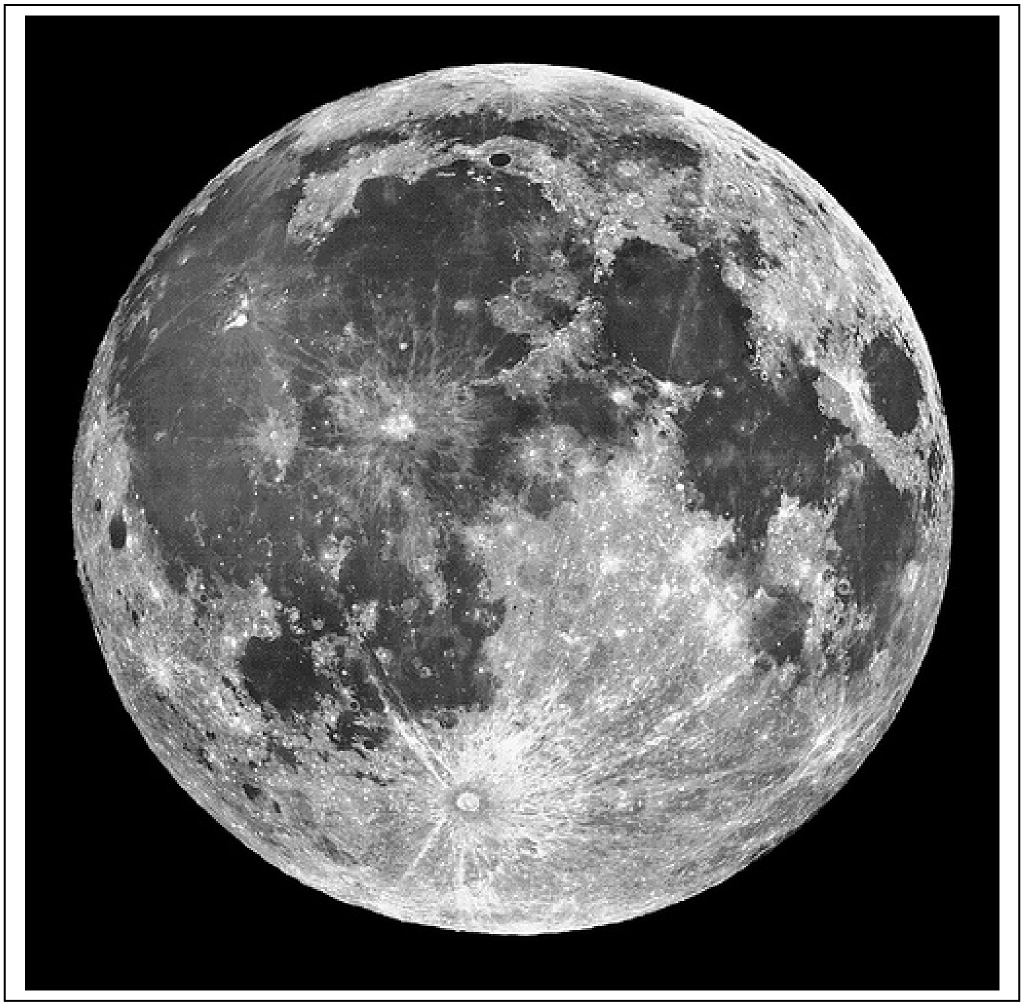
The Moon is truly one of those celestial bodies that needs no introduction! We are all experts in observing it, noting its changing phases, and understanding its 'moonthly' motions in the sky!

As a topic for mathematical study, however, it offers many opportunities to combine both very simple, and very advanced, mathematics topics to further probe its many mysteries. For thousands of years, simple addition, subtraction, multiplication and division was all that was needed to master its chronology in the sky. The advent of telescopes in the 1600's, and the Space Age in the 1960's, however, opened up many new ways to learn about it as a mathematical object.

This booklet is part of a growing collection of mathematics problems for K12 students that explore many different aspects of the moon, from its craters to its thin atmosphere; from its warm interior to its outermost limits in space.

In 2009, NASA launched the Lunar Reconnaissance Orbiter to photograph the lunar surface. It follows a growing international armada of spacecraft that are taking a closer look at the moon in preparation for humans to eventually return and set-up housekeeping. This will require using all of the available lunar resources, which Apollo astronauts discovered and catalogued between 1969-1972 during their historic visits. The search for water ice, and its discovery, would save us billions of dollars in having to drag our own water from Earth to support colonists. The lunar rocks will be chemically processed to extract the ingredients for 'home made' lunar rocket fuel for return journey to Earth, or even outbound trips to Mars!

This booklet will be your guide to some of the mathematical aspects of studying the moon, and eventually sending humans there to live and work.



It is worth reminding students that humans have landed and walked on the moon. This happened during the US Apollo Program between 1969-1972.

Neil Armstrong	Apollo 11	July 21, 1969
Buzz Aldrin	Apollo 11	July 21, 1969
Pete Conrad	Apollo 12	November 19-20, 1969
Alan Bean	Apollo 12	November 19-20, 1969
Alan Shepard	Apollo14	February 5-6, 1971
Edgar Mitchell	Apollo 14	February 5-6, 1971
David Scott	Apollo 15	July 31 - August 2, 1971
James Irwin	Apollo 15	July 31 - August 2, 1971
John Young	Apollo 16	April 21 - 23, 1972
Charles Duke	Apollo 16	April 21 - 23, 1972
Eugene Cernan	Apollo 17	December 11-14, 1972
Harrison Schmidt	Apollo 17	December 11-14, 1972

The Earth and Moon to Scale

We have all seen drawings or sketches in books that show the earth and moon together in the same view, but in reality they are really very different in size, and are much farther apart than you might think.

By creating properly scaled drawings, you will get a better idea of what their sizes are really like! All you will need is a compass, a metric ruler, and a calculator.

The photo above was taken by the Voyager 1 spacecraft on September 18, 1977 at a distance of 7 million miles from Earth, and it has not been edited in any way. Are their diameters to scale? Their distance from each other? Even actual images can be distorted because of perspective and distance effects.

Problem 1 - The radius of the Moon is 1,737 kilometers, and the radius of Earth is 6,378 kilometers. What is the ratio of Earth's radius to the Moon's?

Problem 2 - To the nearest whole number, about how many times bigger than the Moon is Earth?

Problem 3 - With your ruler and compass, draw two circles that represent this size difference, and use a radius of 1 centimeter for the moon disk. Inside the circles, label them 'Earth' and 'Moon'.

Problem 4 - The distance between the center of Earth and the Moon is 384,000 kilometers. To the nearest integer, how many times the radius of Earth is the distance to the Moon?

Problem 5 - Cut out the circles for Earth and the Moon from Problem 3. Using the radius of your circle for Earth as a guide, how far apart, in centimeters, would you have to hold the two cut-outs to make a scale model of the Earth-Moon system that accurately shows the sizes of the two bodies and their distance?

Problem 6 - Look through books in your library, or use GOOGLE to do an image search. Do any of the illustrations show the Earth-Moon system in its correct scale? Why do you think artists draw the Earth-Moon system the way that they do?

Answer Key

Problem 1 - 6378 / 1737 = **3.7.**

Problem 2 - 3.7 is closest to 4.0, so Earth is about **4 times bigger** than the Moon in size.

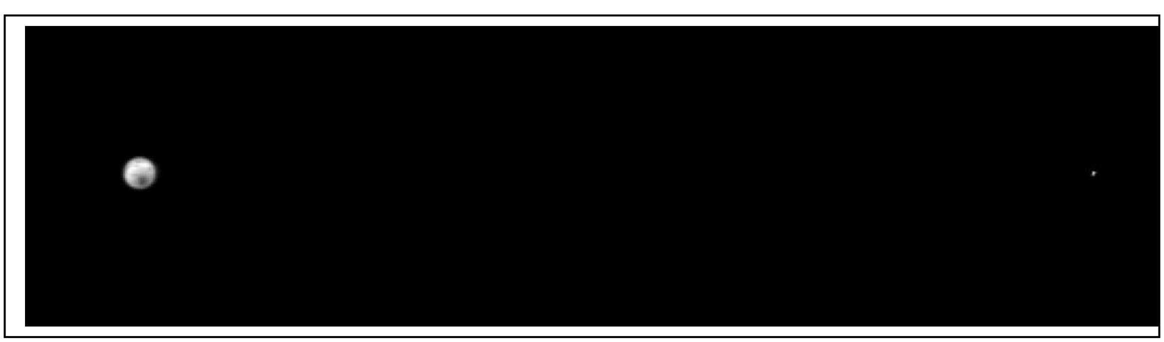
Problem 3 - Draw the disks on a separate paper, but make sure that the Moon has a 1 cm radius and Earth has a 4 cm radius.

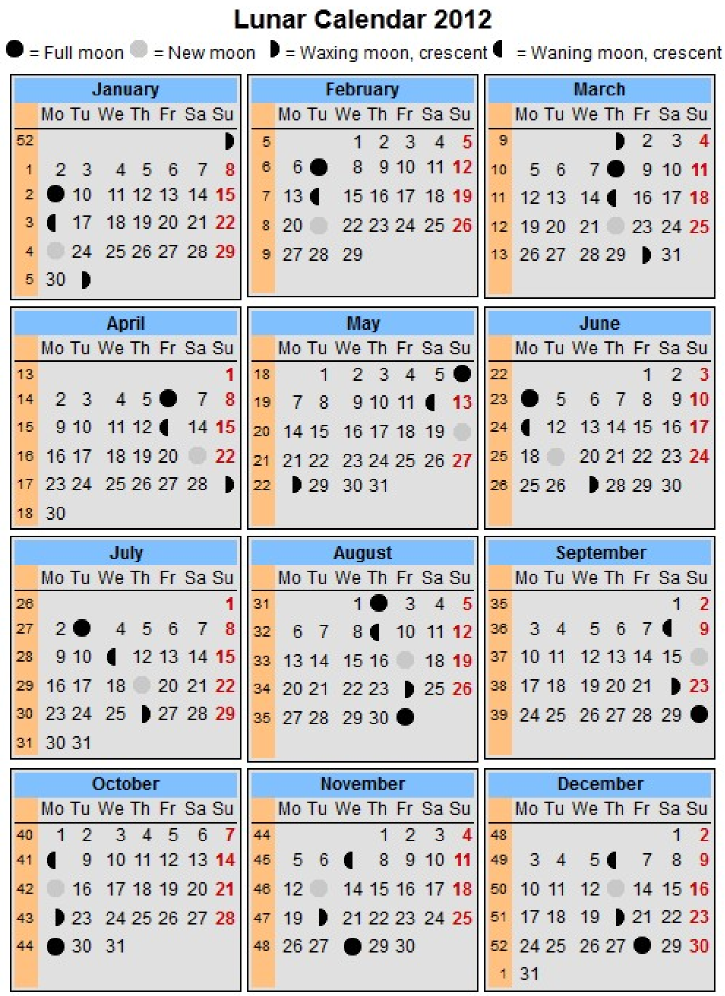
Problem 4 - 384,000 / 6378 = **60.2** which is 60 times Earth's radius.

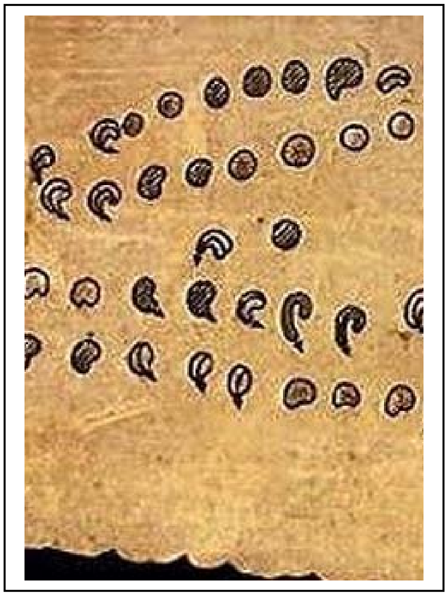
Problem 5 - If the radius of the Moon disk was 1 centimeter, the Earth disk would be 4 centimeters in radius. The distance to the Moon would be 60 times this distance, or 4 cm x 60 = **240 centimeters (2.4 meters)**.

Problem 6 - Very few. Artists try to show a vast 3-dimensional image in a flat perspective drawing that is only a few inches across on the printed page. To draw the Earth-Moon system in the proper perspective scale, the Moon would be a small dot. Also, illustrations that show the phases of the Moon, or eclipses, are also badly out of scale most of the time, because you can't show the phases clearly if the Moon is only the size of a small dot in the illustration. There are other purely artistic reasons too!

Note: The image below was taken by the Mars Odyssey spacecraft soon after launch in April 2001 when Earth and Moon were at their maximum separation. The image is not edited, and shows the disks at their true separations. From the diameter of Earth (12800 km) and its measured diameter in millimeters (4.5 mm) the scale of the image below is 12800/4.5 = 2840 km/mm. The separation from the center of Earth to the Moon 'dot' is 126.5 mm or 126.5 mm x 2840 km/mm = 359,000 km, which is close to its distance according to the US Almanac for April 2001 (…. Km). However, although Earth is 3.7x bigger than the Moon, it is clear that the lunar dot, which measures just under 0.5 mm) is much smaller than it should be (diameter = 3,474 km or 1.2 mm). **Compared to the Earth-Moon distance, how far from the Moon was the Mars Orbiter in order to see the Moon with this disk diameter? Answer: About 359,000 km x 1.2/0.5 = 860,000 km.**







The archaeological record's earliest data that speaks to human awareness of the stars and 'heavens' dates to the Aurignacian Culture of Europe, c.32,000 B.C during the Late Upper Paleolithic period of Europe. Archeologists have deciphered sets of marks carved into animal bones, and occasionally on the walls of caves, as records of the lunar cycle. An example is shown on the right.

Problem 1 – From the calendar for 2012, and to the nearest tenth of a day, what is the average number of days between consecutive full moons?

Problem 2 – What is the maximum number of full moons can you have in a given year?

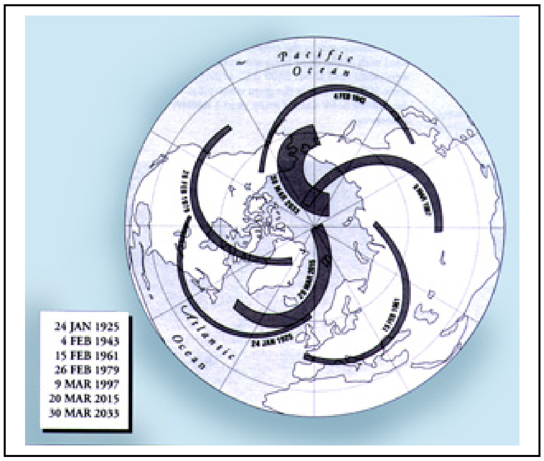
Problem 1 – From the calendar for 2012, and to the nearest tenth of a day, what is the average number of days between consecutive full moons?

Answer: Starting on January 9, the number of days between the full moons in 2012 is 29, 30, 29, 30, 29, 29, 30, 29, 30, 29, 30, 30. There are 12 intervals, so the average time of the interval is 354/12 = **29.5 days.**

Problem 2 – What is the maximum number of full moons can you have in a given year?

Answer: If the first full moon of the year is on January 1, then for an average time interval of 29.5 days, you can have a total of 365/29.5 = 12 full moon intervals, so counting from the first full moon on Janunary 1, there can be a total of **13**, with the last full moon occurring 11 or 12 (Leap year has 366 days) days before the end of the year

Note that because many months have 30 and 31 days, you can have two full moons on the same month, for example August 2012. The second full moon is sometimes called a Blue Moon!



Since earliest recorded history, humans have known about the lunar month, and many societies still use a lunar calendar rather than a solar calendar. It is also known that ancient astronomers didn't just stop with a knowledge of the 29.5-day lunar month, but also realized that the moon was involved with dramatic solar eclipses, and that solar eclipses followed a pattern that also depended on the lunar month. You can only have total solar eclipses during New Moon, and you can only have lunar eclipses during a Full Moon. This led to the discovery in Ancient Greece of an additional cycle called the Saros or the Metonic Cycle.

Problem 1 – Assume that the time between lunar phases is exactly 30 days, and that a year is exactly 400 days long. Use the method of finding the Least Common Multiple to determine how many years have to elapse before the same lunar phase occurs on the same day of the year?

Problem 2 – The Saros period is the time between lunar (or solar) eclipses on Earth. For example, if we start with New Moon and a total solar eclipse and add one Saros period, we come to a second New Moon and a second total solar eclipse. Suppose that the time between full moons is exactly 29 days. How many years of exactly 365 days have to elapse before the total number of full moons is an integer number?

Problem 3 – The figure above shows a series of total solar eclipses and their dates. About how many years separate the eclipses in this Saros Series?

Answer Key

Problem 1 – Assume that the time between lunar phases is exactly 30 days, and that a year is exactly 400 days long. Use the method of finding the Least Common Multiple to determine how many years have to elapse before the same lunar phase occurs on the same day of the year?

Answer: We need M x 30 = N x 400.

Prime factors: 30 = 3 x 2 x 5 and 400 = 2 x 2 x 2 x 2 x 5 x 5

The LCM = 3 x 24 x 52 = 1200 days which is N = 3 and M = 40
so we have to wait **3 years.**

Problem 2 – The Saros period is the time between lunar (or solar) eclipses as viewed from the same spot on Earth. For example, if we start with New Moon and a total solar eclipse and add one Saros period, we come to a second New Moon and an almost identical total solar eclipse. Suppose that the time between full moons is exactly 29 days. How many years of exactly 365 days have to elapse before the total number of full moons is an integer number?

Answer: We need M x 29 = N x 365 where M and N are integers.

Prime Factors: 29 = 29 365 = 5 x 73 then LCM = 5 x 29 x 73 = 10585 days, then M = 365 lunar cycles

and we have to wait **N = 29 years.**

Note: A more accurate result gives 223 x 29.53 days = 18.03 x 365.25 days, so a Saros cycle lasts 18.03 = 18 years plus 11 days and has exactly 223 complete lunar cycles.

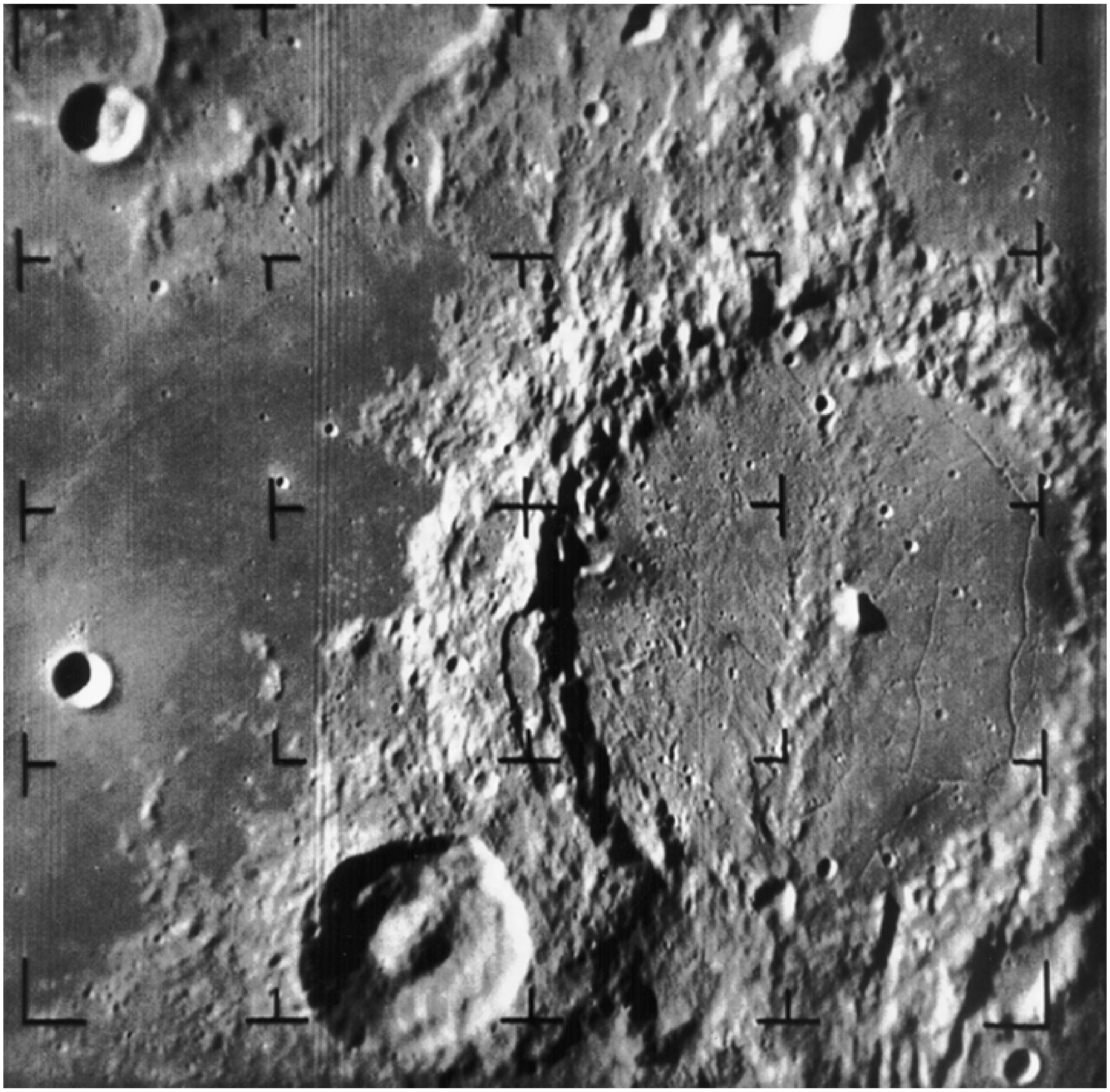
223 x 29.53 = 6585.2 and 18.03 x 365.25 = 6585.5 days

Problem 3 – The figure above shows a series of total solar eclipses and their dates. How many years separate the eclipses in this Saros Series?

Answer: Answer: The pairs of eclipses are separated by 18 years. In addition, the month and day dates are separated by about 11 days, which matches the Saros period of 18.03 years or 18 years 11 days.

The Moon Closeup - I

In 2009, NASA's Lunar Reconnaissance Orbiter (LRO) will orbit the moon and take high-resolution images of the lunar surface. This will be the beginning of the 'return to the moon' program that will lead to astronauts landing on the moon sometime between 2018 and 2025. The image below shows the Alphonsus crater from a distance of 442 km, taken about three minutes before impact of the Ranger 9 spacecraft on 24 March 1965 at 14:08:20 UT.



Problem 1 - Using a millimeter ruler, what is the scale of this image in kilometers / millimeter if the width of this picture is 183 kilometers?

Problem 2 - What is the diameter of the crater Alphonsus in kilometers?

Problem 3 - What is the size of the smallest thing you can see in the picture, in meters?

Problem 4 - How wide are the channels inside the crater Alphonsus?

Problem 5 - Where do you think the safest place would be to land in this image?

Answer Key

Problem 1 - Using a millimeter ruler, what is the scale of this image in kilometers / millimeter if the width of this picture is 183 kilometers?

Answer: The image measures 153 millimeters wide, so if this equals 183 kilometers, then the scale of the image is 183 km/153 mm = 1.2 kilometers/millimeter.

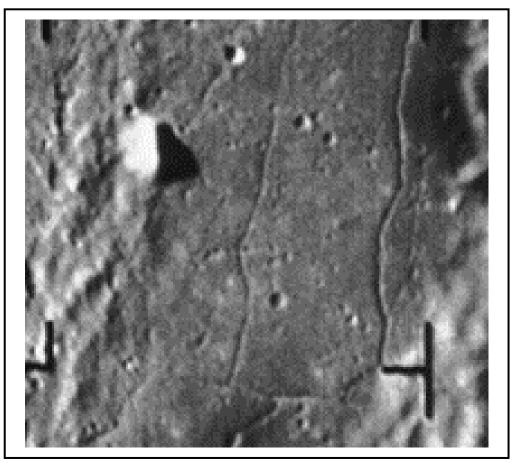
Problem 2 - What is the diameter of the crater Alphonsus in kilometers?

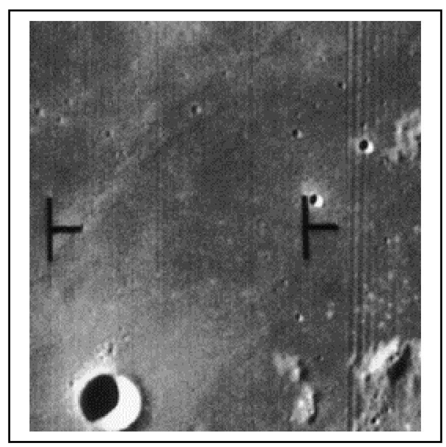
Answer: The diameter is about 85 millimeters ,which from the image scale equals 85 mm x 1.2 km/mm = 102 kilometers. Students may have a discussion about where to measure the edge of the crater. If you select the outer-most edge of the crater wall, the diameter is closer to 110 millimeters so the diameter is 110 x 1.2 = 132 kilometers.

Problem 3 - What is the size of the smallest thing you can see in the picture, in meters?

Answer: By carefully looking at the printed image, the smallest craters are about 0.5 millimeters across. This equals 0.5 mm x 1.2 km/mm = 0.6 km or 600 meters.

Problem 4 - How wide are the channels inside the crater Alphonsus?





Answer: The channels, which you can see in the image above (left) probably created by faults or collapsed lava tubes, are about 0.5 millimeters wide which equals 600 meters. The longest channel is about 73 millimeters long or 88 kilometers.

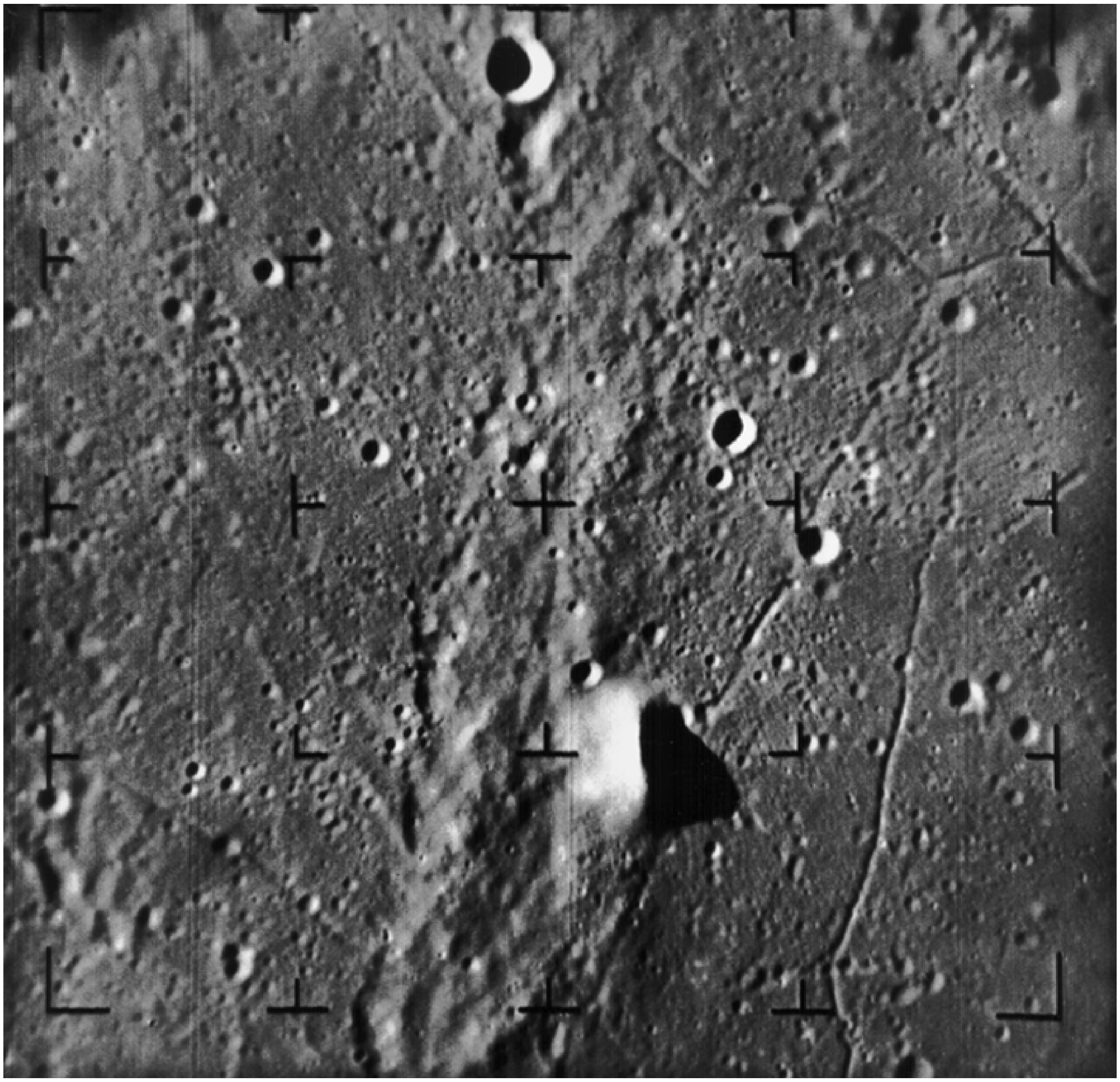
Problem 5 : Where do you think the safest place would be to land in this image?

Answer: The safest place is where there are the fewest features that you can see in the photograph above (right). This is in the dark 'basin' areas to the left of the crater where you can't see any rough spots at the scale of this image.

The Lunar Reconnaissance Orbiter Camera (LROC) will take high-resolution pictures of places like this on the moon. The Lunar Orbiter Laser Altimeter (LOLA) will gather topography data to aid in the search for large boulders that may pose a safety and landing hazard for future astronauts.

The Moon Closeup - II

The Lunar Reconnaissance Orbiter (LRO), to be launched in 2009, will create an atlas of high-resolution images of the lunar surface to examine potential landing sites for future astronauts. This is a NASA, Ranger 9 image taken 54 seconds before impact. The upraised area at lower center is the central peak of Alphonsus crater floor. This image was taken from a distance of 136 km.



Problem 1 - If the image is 55 kilometers across, use a millimeter ruler to determine the scale of the image in meters/millimeter.

Problem 2 - What is the smallest feature you can find in the image? How does it compare to the size of your house?

Problem 3 - The Washington DC beltway is a highway (495) with a diameter of 26 kilometers that encircles 3 million people, and over 100,000 homes and buildings. How big would this area be if it were drawn to the same scale as the image above?

Answer Key

Problem 1 - If the image is 55 kilometers across, use a millimeter ruler to determine the scale of the image in meters/millimeter.

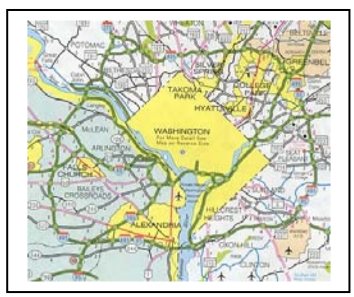
Answer: The width of the photo is 152 millimeters so the scale will be 55 kilometers/152 millimeters = 0.360 km/mm or 360 meters/millimeter.

Problem 2 - What is the smallest feature you can find in the image? How does it compare to the size of your house?

Answer: The smallest craters are about 1 millimeter wide or 360 meters. A typical house is about 50-feet wide or 17 meters, so these small craters are about 360/17 = 21 times bigger than a house.

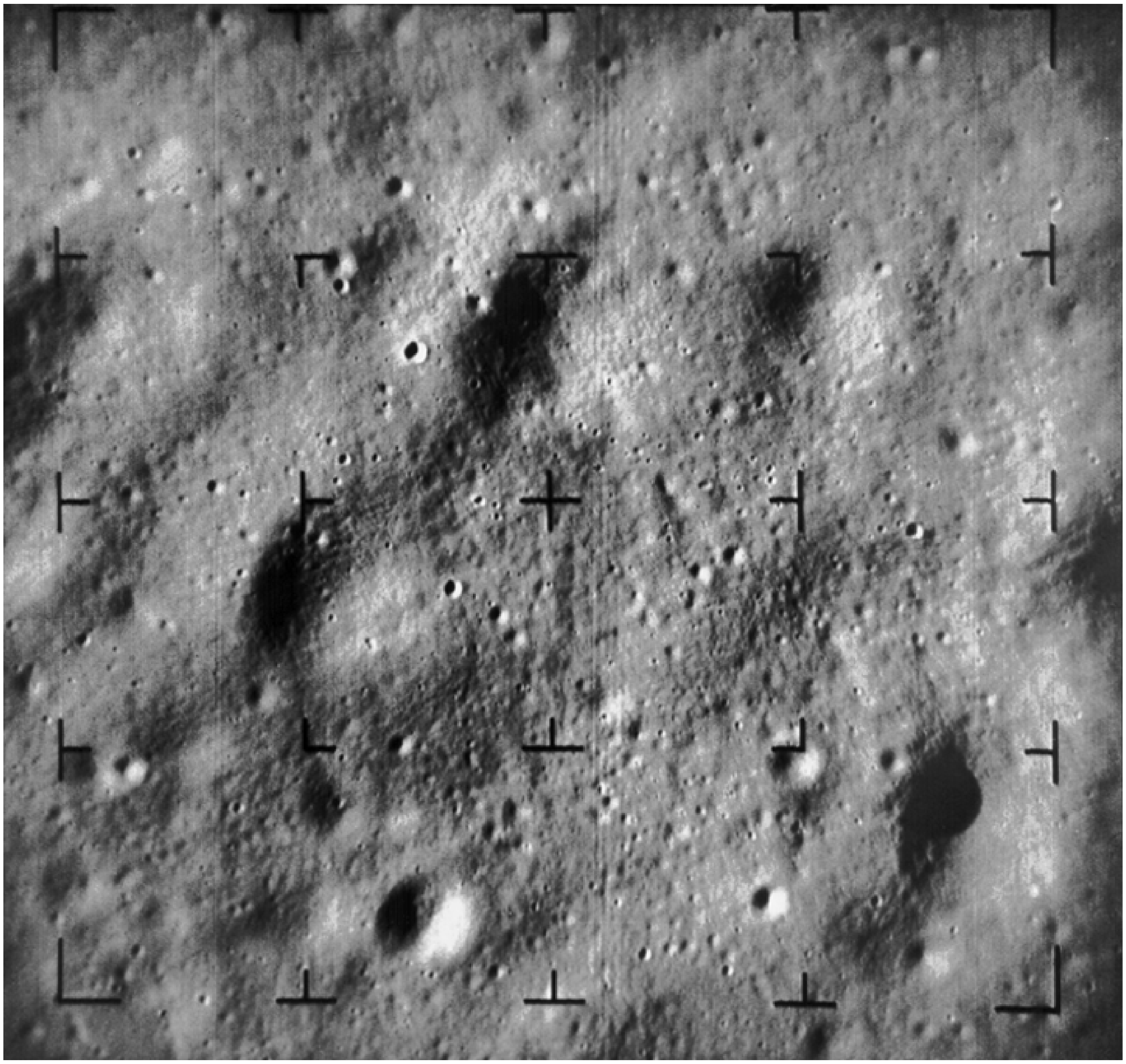
Problem 3 - The Washington DC beltway is a highway (495) with a diameter of 26 kilometers that encircles 3 million people, and over 100,000 homes and buildings. How big would this area be if it were drawn to the same scale as the image above?

Answer: At the scale of the lunar photo, a circle with a diameter of 26 kilometers would equal 26 km / (0.36 km/mm) = 72 millimeters. This is about 1/2 the width of the photo. The photos below give an idea of the relative sizes at about the same scale.



The Moon Closeup - III

In 2009, NASA's Lunar Reconnaissance Orbiter (LRO) will orbit the moon and create a high-resolution atlas of its surface for future astronauts. The image below is the last one taken by NASA's Ranger 9 wide-angle camera about three seconds before impact on March 24, 1965. It shows the floor of Alphonsus crater from an altitude of 7.5 kilometers. LRO cameras will take pictures far more detailed than this!



Problem 1 - If the image is 3.3 kilometers across, what is the scale of the image in meters/millimeter?

Problem 2 - What is the smallest crater you can find in the image? How big is it compared to your house?

Problem 3 - If you were planning to land a spacecraft, what area might you select as the safest place to avoid hitting boulders or craters that are 30-meters across?

Problem 1 - If the image is 3.3 kilometers across, what is the scale of the image in meters/millimeter?

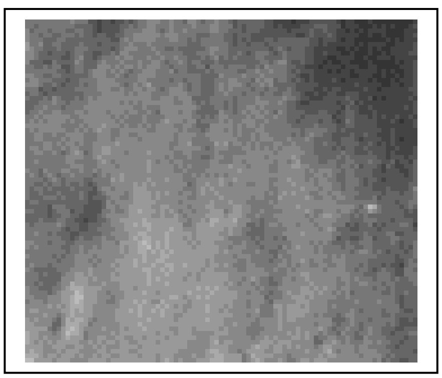
Answer: The width of the image is 153 millimeters, so the scale is 3.3 km/153 mm = 0.0216 km/mm or 21.6 meters/mm.

Problem 2 - What is the smallest crater you can find in the image? How big is it compared to your house?

Answer: A careful study of the image shows small craters and pits that are about 0.5 millimeters across, which correspond to 0.5 mm x 21.6 m/mm = 10.7 meters in diameter.

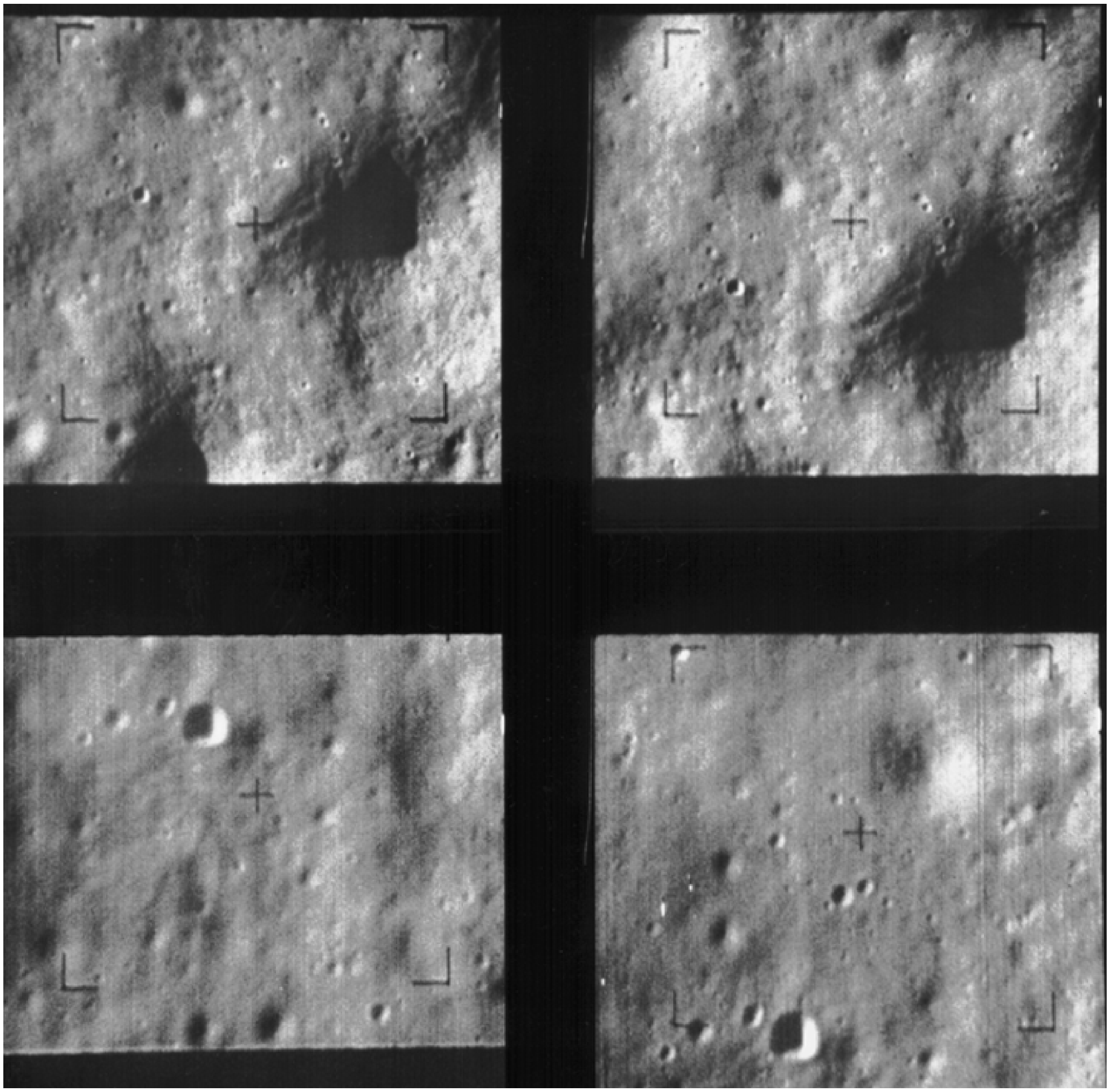
Problem 3 - If you were planning to land a spacecraft, what area might you select as the safest place to avoid hitting boulders or craters that are 30-meters across?

Answer: You would want to find places in the photo where you can't see any pits or details that show up at a scale of 1.0 to 0.5 millimeters across. These would look very smooth to the eye on the photo. Here is one spot that is located in the upper right corner of the picture. There are many other areas just like it. The picture below is about 200 meters on a side. Have your students compare this to a familiar object of the same size. A suggestion is a large city block, a downtown shopping mall, or a park.



The Moon Closeup - IV

These four images of the floor of Alphonsus crater were taken from an altitude of about 14 kilometers on 24 March 1965 five seconds before NASA's Ranger 9 spacecraft impacted. The pictures were taken by the cameras (clockwise from upper left) P3, P4, P2, P1. The P3 image 1600 m across. The P4 image is 1500 km across. P2 is 540 meters across, and P1 is 580 meters.



Problem 1 - What are the image scales, in meters/mm, of each of the four images?

Problem 2 - What is the smallest feature that was seen by any of the four cameras?

Problem 3 - What is the location of the P2 image in the picture taken by P3? Draw a box in P3 of the correct size.

Problem 4 - Draw a box in the image used in the problem 'Lunar8' that shows the field of P3. How many common features can you identify?

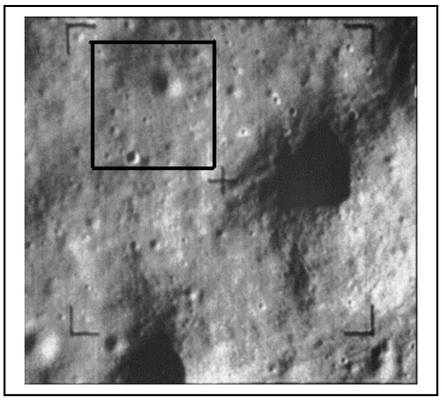
Problem 5 - Compare P1 to the scale of 'Lunar6'. By what magnification does P1 represent of the field in 'Lunar1'?

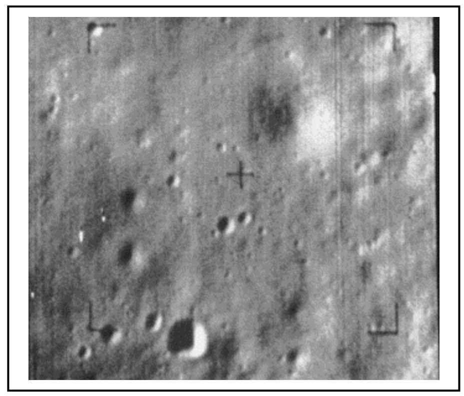
Answer Key

Problem 1 - What are the image scales of each of the four images? Answer: The images are all 71 millimeters across, so the image scales are P3 = 1600 m/71mm = 22.5 m/mm; P4 = 1500 m/71mm = 21.1 m/mm; P2 = 540 m/71 mm = 7.6 m/mm and P1 = 580 m/71mm = 8.2 m/mm.

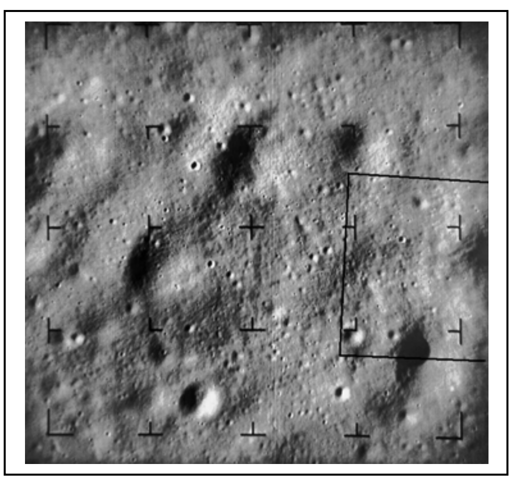
Problem 2 - What is the smallest feature that was seen by any of the four cameras? Answer: The image with the highest scale is the P2 field. The smallest features are 1 mm or 7.6 meters across.

Problem 3 - What is the location of the P2 image in the picture taken by P3? Draw a box in P3 of the correct size. Answer: The size of P2 is 71 mm x 63 mm which for a scale of 7.6 m/mm = 540m x 479m. At the scale of the P3 image of 22.5 m/mm, the field of P3 would be 24 mm x 21 mm. The location is shown below:



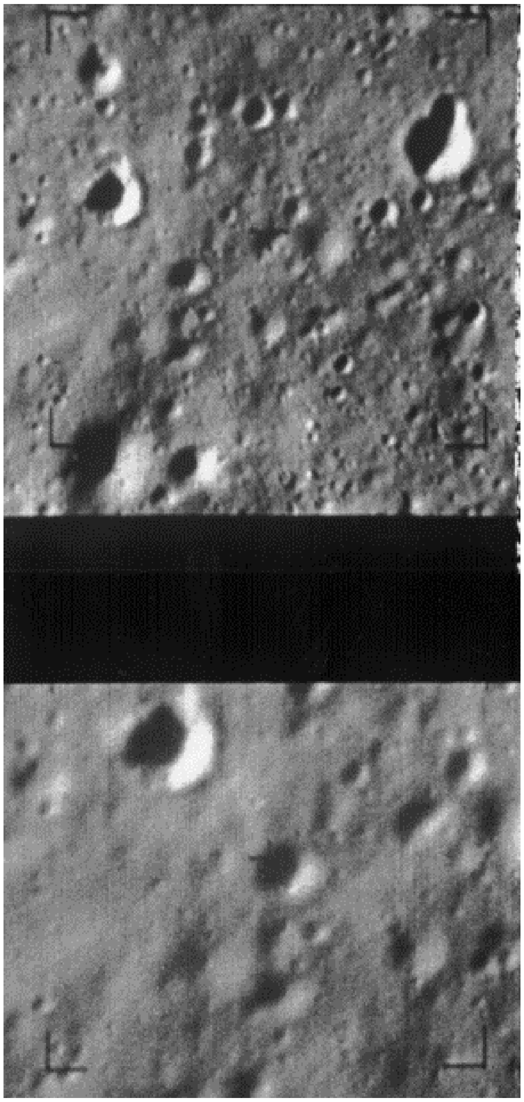


Problem 4 - Draw a box in the image used in the problem 'Lunar8' that shows the field of P3. How many common features can you identify? Answer: See below for the approximate area.



Problem 5 - Compare P1 to the scale of the image of the Alphonsus Crater region in 'Lunar6'. By what magnification does P1 represent of the field in 'Lunar1'? Answer: The scale of 'Lunar6' image is 1.2 kilometers/mm while the scale of the P1 image is 8.2 meters/mm. This means that the P1 image compared to Lunar6 has been magnified by 1200m/8.2m = 14.6 times.

The Moon Closeup - V



This image shows the last two pictures taken by NASA's Ranger 9 about 0.25 seconds before impact onto the lunar surface on March 24, 1965. The top image, taken at an altitude of 600 meters above the lunar surface, was taken by camera P3 and is about 70 m across. The lower frame is from camera P1 and is approximately 50 meters across.

Problem 1 - How fast was the Ranger 9 spacecraft traveling just before it struck the lunar surface: A) In kilometers/sec? B) In kilometers/hour? C) In miles/hour?

Problem 2 - What is the scale of A) the top image, P3, in meters/millimeter? B) the bottom image, P1, in meters/mm?

Problem 3 - What is the smallest feature you can see in the P1 image, and how large is it compared to familiar things you know about (cars, houses, people, etc)?

Problem 4 - The Lunar Reconnaissance Orbiter (LRO) has a high-resolution camera called the LROC capable of seeing details as small as 0.5 meters per pixel. How many millimeters across would such features appear on the P1 image?

Problem 5 - Draw a box in the P3 image that corresponds to the field covered by the P1 image using the information about the scales of the two images, the size of the P1 image in millimeters, and common features that appear in each photo.

Problem 6 - Using the P1 image as a guide, draw some familiar things that you know about to the same scale and place them in the photograph.

Answer Key

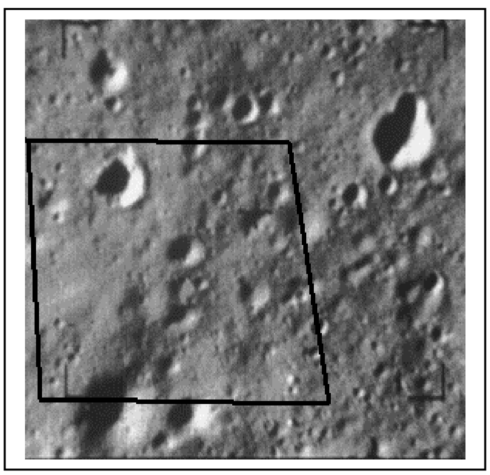
Problem 1 - How fast was the Ranger 9 spacecraft traveling just before it struck the lunar surface: A) In kilometers/sec? B) In kilometers/hour? C) In miles/hour? Answer: in 0.25 seconds, the spacecraft traveled 800 meters. A) 0.8 km/0.25 sec = 3.2 km/sec. B) 3600 sec/hr x 3.2 km/sec = 11,520 km/hour. C) 0.62 km/mile so 11,520 km/hr / (0.62 km/mile) = 18,580 miles/hour.

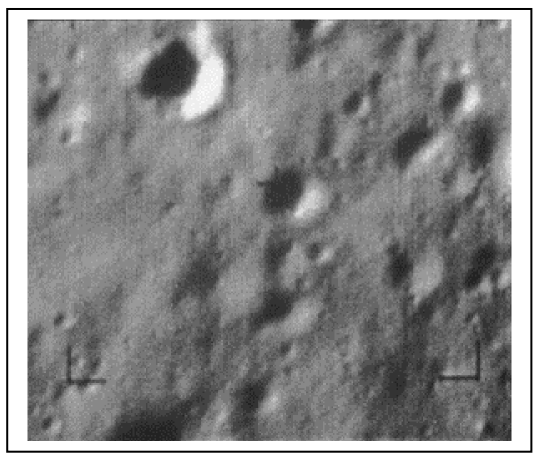
Problem 2 - What is the scale of A) the top image, P3, in meters/millimeter? B) the bottom image, P1, in meters/mm? Answer: A) 70 meters/62 mm = 1.1 meters/mm. B) 50 meters/65 mm = 0.77 meters/mm or 77 cm/mm.

Problem 3 - What is the smallest feature you can see in the P1 image, and how large is it compared to familiar things you know about (cars, houses, people, etc)? Answer: Smallest features are about 1 millimeter or 77 centimeters across on the lunar surface. This is about as large as a large dog or a bush.

Problem 4 - The Lunar Reconnaissance Orbiter (LRO) has a high-resolution camera called the LROC capable of seeing details as small as 0.5 meters per pixel. How many millimeters across would such features appear on the P1 image? Answer; In the P1 image, a feature 0.5 meters across would be about 0.5 / 0.77 = 0.6 millimeters across which is very close to the smallest feature we can see in the P1 image.

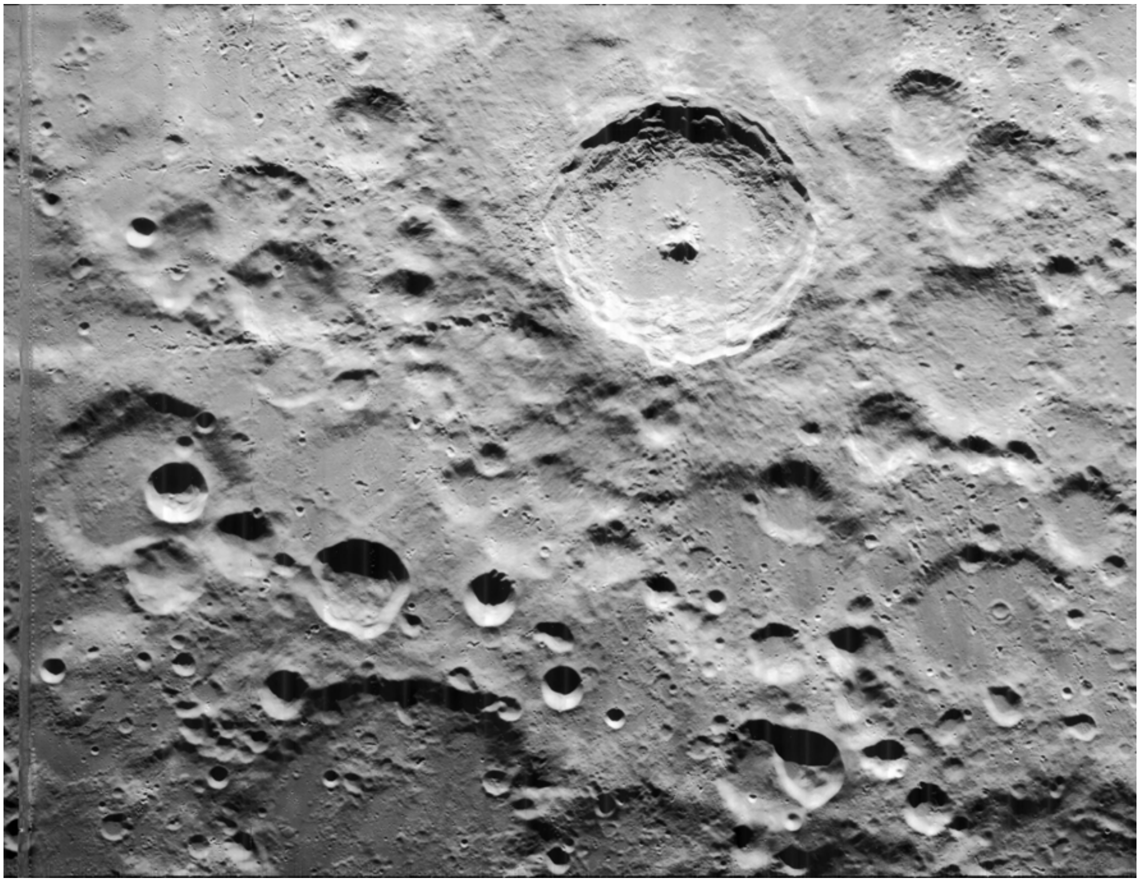
Problem 5 - Draw a box in the P3 image that corresponds to the field covered by the P1 image using the information about the scales of the two images, the size of the P1 image in millimeters, and common features that appear in each photo. Answer; The P1 image is 50 meters wide by 41 meters tall. At the 1.1 m/mm scale of the P3 image, this would be a box with dimensions 45mm x 37 mm. There are several large craters in common that make it easy to 'register' the two images as shown below (left).





Problem 6 - Using the P1 image as a guide, draw some familiar things that you know about to the same scale and place them in the photograph. Answer: See image to the right for an example of houses and streets at about the same scale as the P1 image (top right).

Craters on the Moon - I



This is a NASA image taken by the Lunar Orbiter IV spacecraft as it captured close-up images of the lunar surface in May, 1967. The large crater at the top-center is Tycho. Other images from the Lunar Orbiter spacecrafts can be found at the Lunar Orbiter Photo Gallery (http://www.lpi.usra.edu/resources/lunarorbiter/) The satellite was at an altitude of 3,000 kilometers when it took this image, which measures 350 km x 270 km.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 350 kilometers x 270 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What is the diameter of the crater Tycho in kilometers?
Question 2: How large is the smallest feature you can see?
Question 3: How large are some of the smaller hills at the floor of the crater, in meters?
Question 4: About how large are the most common craters in the field?
Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?

Answer Key:

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?
Answer: 150 millimeters.

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.
Answer: 350 kilometers.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.
Answer: 350 kilometers / 150 millimeters = 2.3 kilometers / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

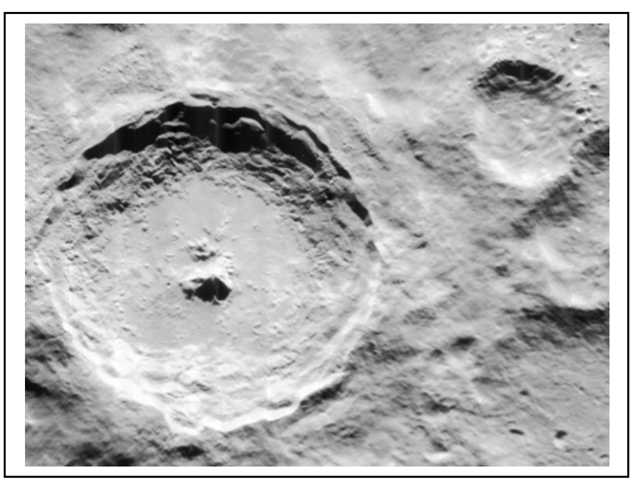
Question 1: What is the diameter of the crater Tycho in kilometers?
Answer: About 35 millimeters x 2.3 km/mm = 80.5 kilometers in diameter which is 80 kilometers to two significant figures.

Question 2: How large is the smallest feature you can see?
Answer: There are many small details in the image, pits, hills, etc, that students can estimate 0.1 to 0.3 millimeters for a physical size of 0.2 to 0.7 kilometers since the measurement is only good to one significant figure.

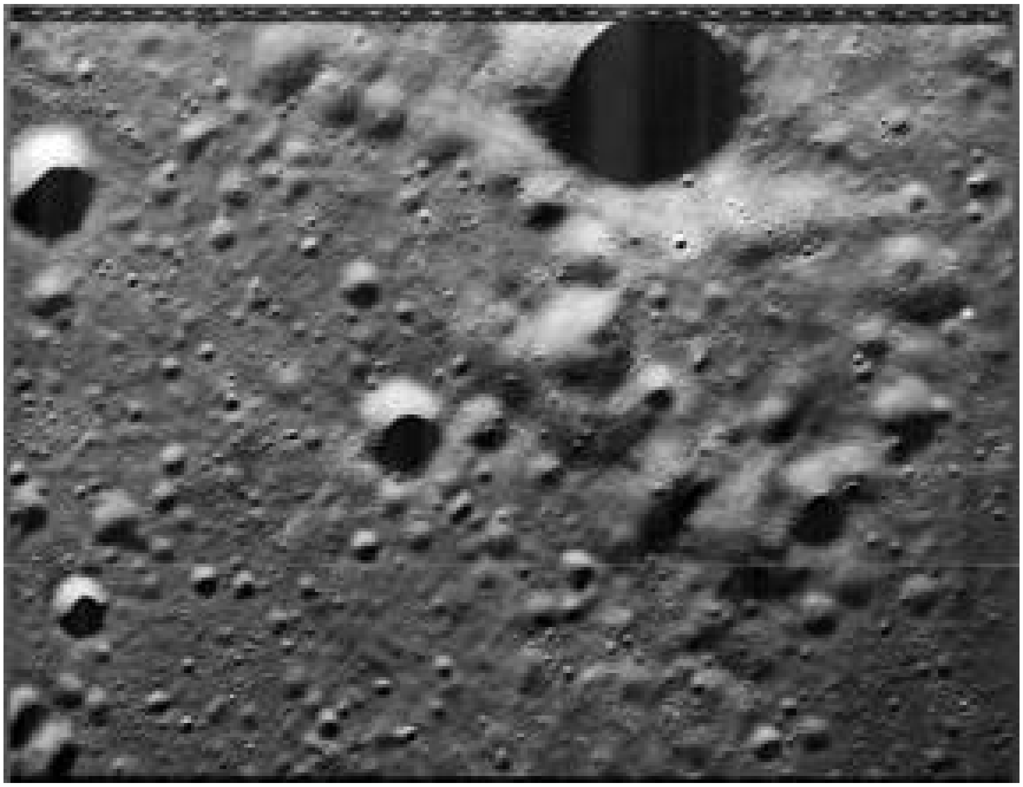
Question 3: How large are some of the smaller hills at the floor of the crater, in meters?
Answer: These small features are about 0.1 millimeters across or 200 meters in size.

Question 4: About how large are the most common craters in the field?
Answer: The answer may vary a bit, but the small craters that are 0.5 millimeters across are the most common. These have a physical size of about 1 kilometer.

Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?
Answer: In order to fit Denver into one of these lunar craters, it will have to appear to be about
25 km x (1.0 millimeter/2.3 km) = 11 millimeters across. There are three caters just to the right of Tycho that are about this big. Students should not get 'lost' trying to exactly match up their estimate with a precise lunar feature. 'Close-enough' estimates are good enough! See below comparison as a guide.



Craters on the Moon - II



This is a high resolution image of the lunar surface taken by NASA's Lunar Orbiter III spacecraft in February 1967 as it orbited at an altitude of 46 kilometers. It is located near the lunar equator. The field of view is 16.6 kilometers x 4.1 kilometers. Additional Orbiter images can be found at the Lunar Orbiter Gallery (http:// www.lpi.usra.edu/resources/lunarorbiter/). Because of the low sun angle, craters look like circles that are half-black, half-white inside!

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

Question 1: How big is the largest crater in the image?
Question 2: How big is the smallest crater in the image, in meters?
Question 3: About what is the typical distance between craters in the image?
Question 4: How far would you have to walk between the largest, and next-largest craters?

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?
Answer: 134 millimeters.

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.
Answer: 16600 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.
Answer: 16600 meters / 134 millimeters = 124 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

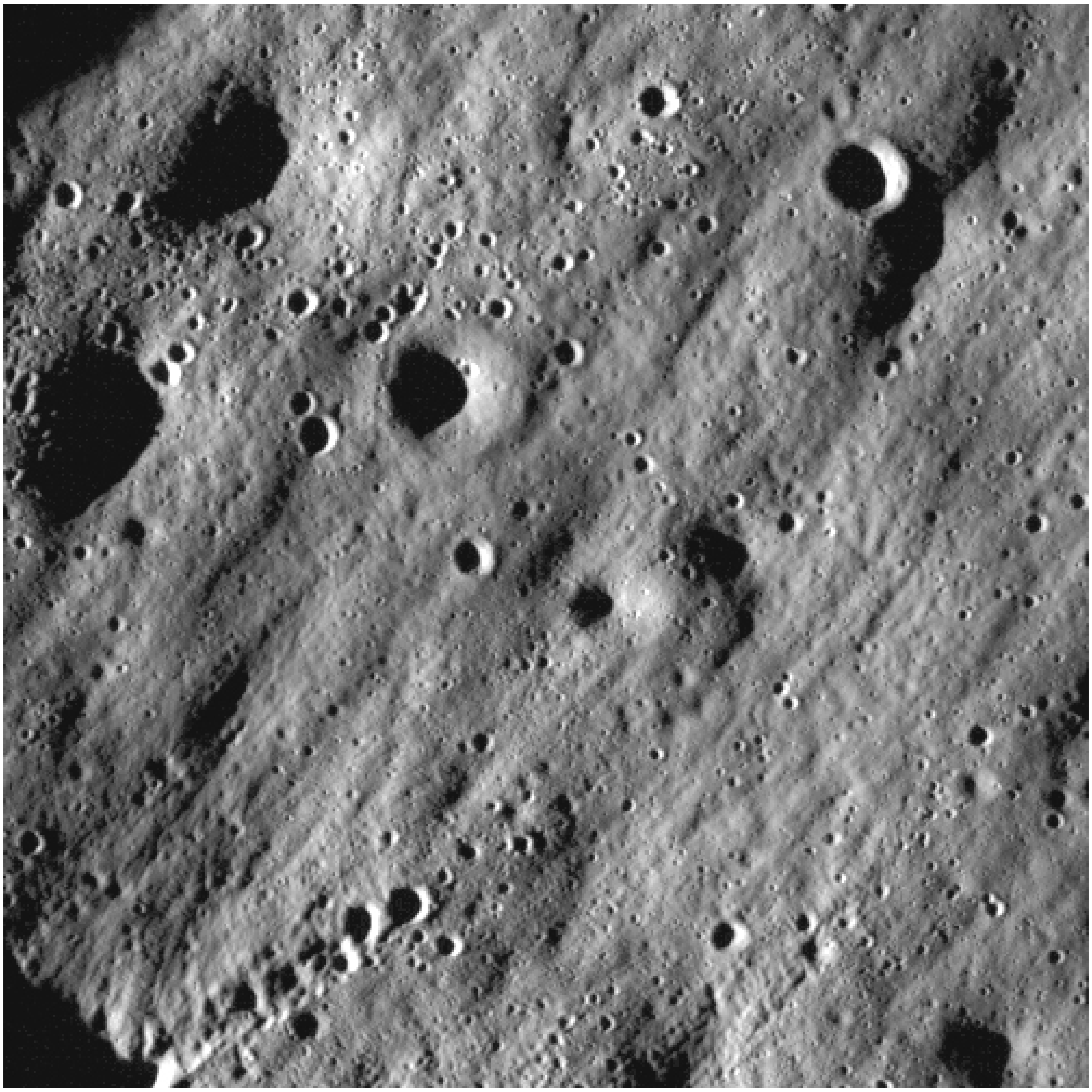
Question 1: How big is the largest crater in the image?
Answer: The one at the top is about 25 millimeters across. 25 mm x 124 meters/mm = 3,100 meters or 3.1 kilometers.

Question 2: How big is the smallest crater in the image, in meters?
Answer: The largest number of features are about 1 millimeter across or 100 meters to one significant figure. Students may also go for the recognizable craters which are about 2 millimeters across or about 200 meters.

Question 3: About what is the typical distance between craters in the image?
Answer: The answer may vary, but the distance between obvious craters (about 2 mm in diameter) is about 5 millimeters or 5 mm x 124 meters/mm = 600 meters to one significant figure.

Question 5: How far would you have to walk between the largest, and next-largest craters?
Answer: The crater rims are about 35 millimeters apart or 35 mm x 124 meters/mm = 4,340 meters or 4.3 kilometers to two significant figures.

LRO's First Image of Mare Nubium



This is one of the first images taken by LRO showing details in Mare Nubium. The width of the image is 700 meters (500 pixels).

Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel.

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find?

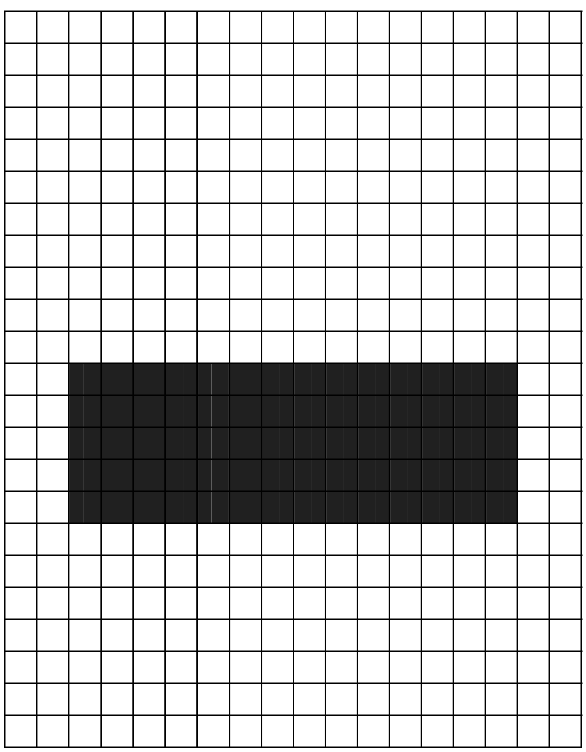
Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. (Assume 1 meter = 3 feet)

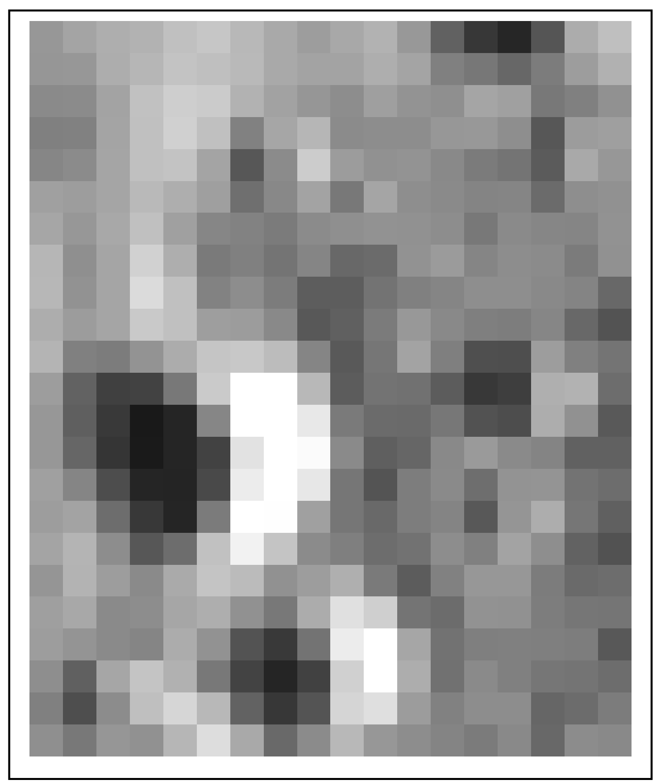
Answer Key

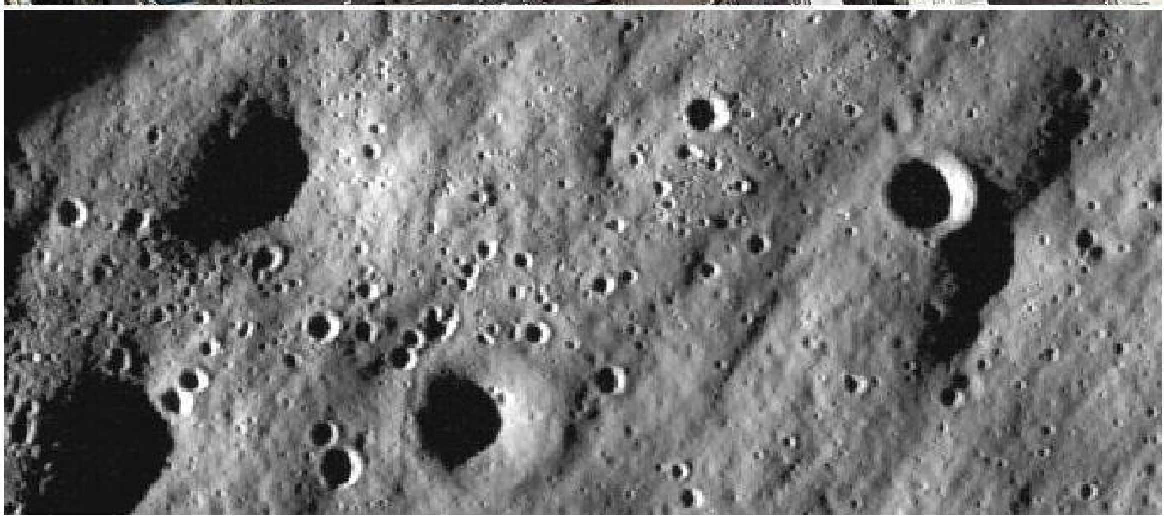
Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel. Answer: Width = 153 millimeters so the scale is 700 meters/153 mm = **4.6 meters/mm**, and 700 meters/500 pixels = **1.4 meters/pixel.**

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find? Answer: Students should see craters as small as 0.5 millimeters or 0.5 mm x 4.6 m/mm = **2.3 meters.**

Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. Answer: First convert the feet into metric units. Three feet equals about 1 meter, so the yard measures 75 feet x 96 feet = 25 meters x 32 meters, and the house measures 7 meters x 20 meters. At the scale of the LRO image of 1.4 meters/pixel, the property is **18 pixels x 23 pixels,** and the house measures **5 pixels x 14 pixels.** See sketch below, and the comparison lunar image enlargement.







The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above 700-meter wide image shows downtown Las Vegas, Nevada (Top - Courtesy of Digital Globe, Inc.), and Mare Nubium (bottom - LRO) at this same resolution.

Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image?

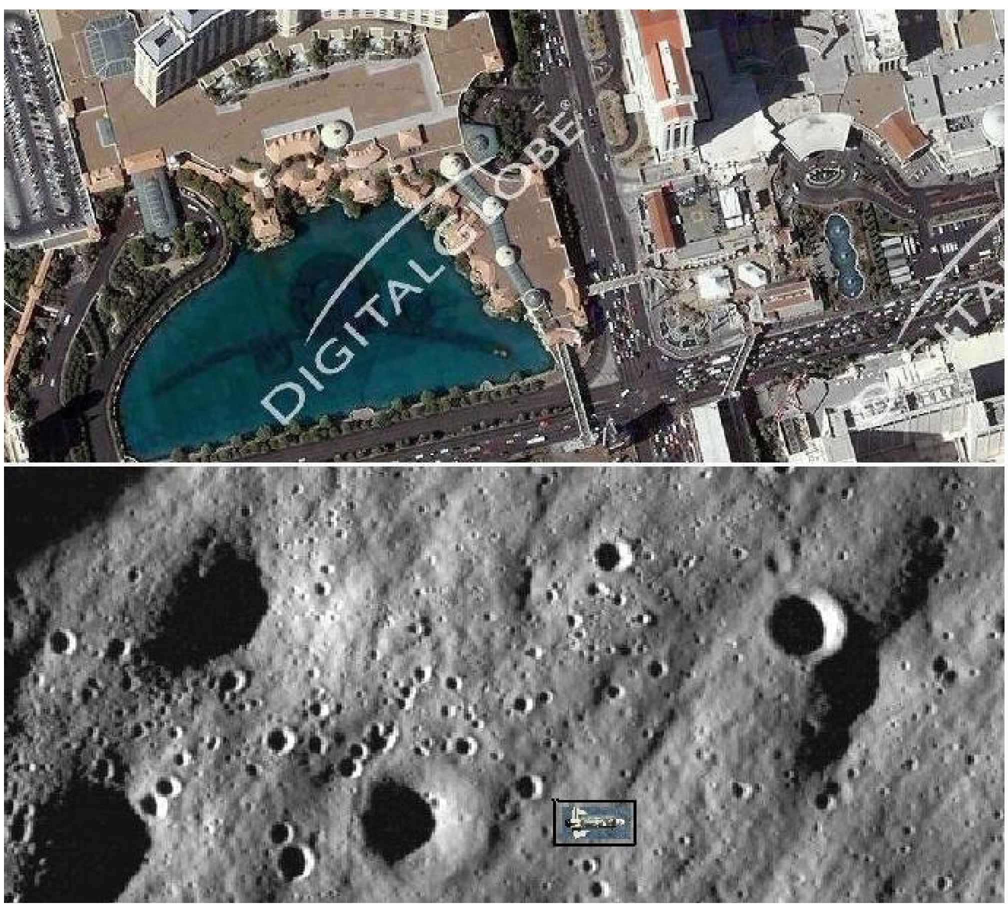
Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood?

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!).

Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image? Answer: the image is 153 millimeters wide so the scale is 700 meters/153 mm = **4.6 meters/mm.** Small craters are about 4-5 meters across; medium craters are about 10 to15 meters across, and the few large craters are about 30 to 100 meters across.

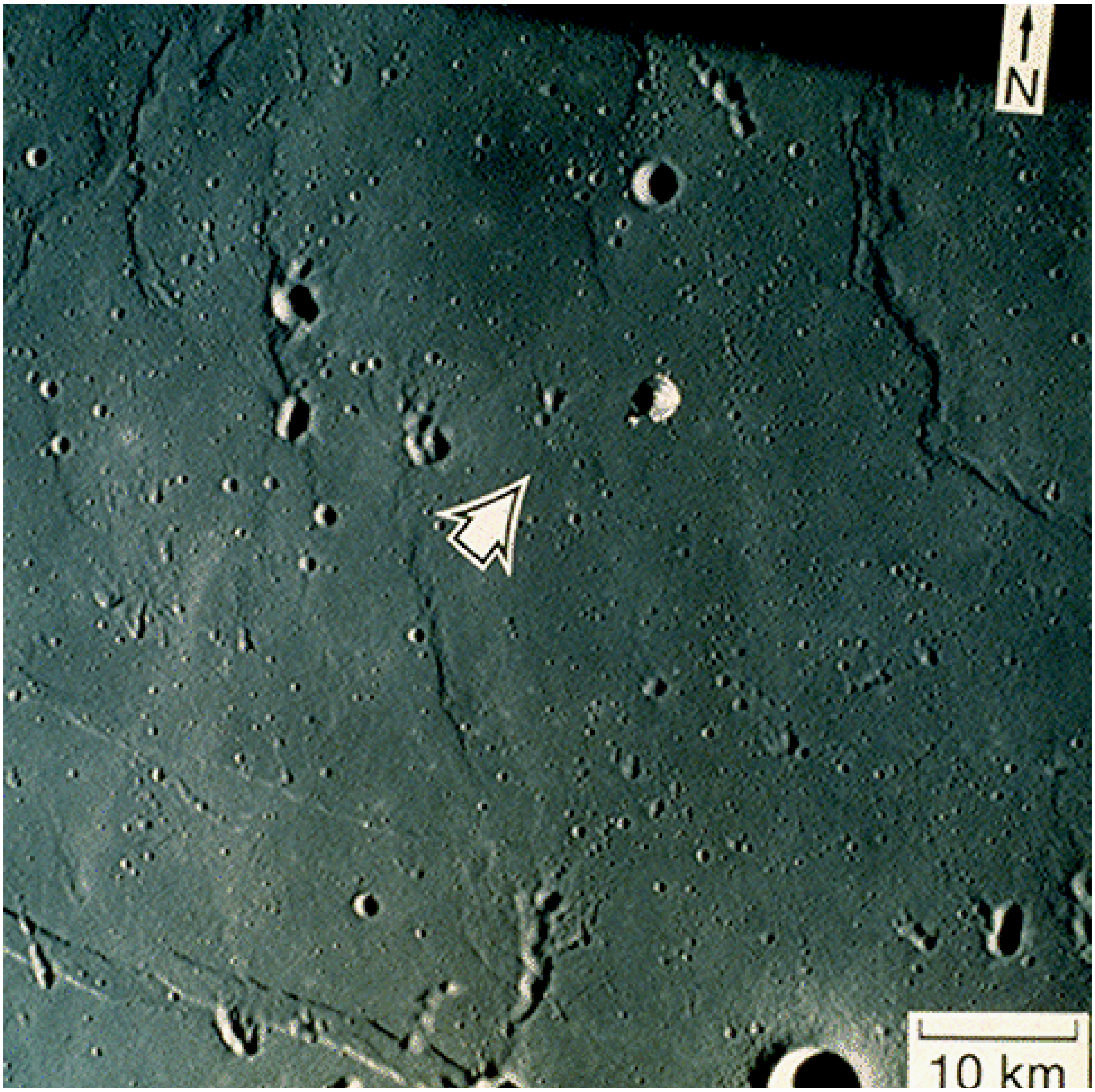
Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood? Answer: The small craters are about as wide as your car, mini-van or street. The medium craters are about as wide as large as your house. The big craters are as big as your entire yard or a large Boulevard.

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!). Answer: **The shuttle would be 37 meters/4.6 M/mm = 8 millimeters long by 24/4.6 = 5.2 millimeters wide**. The figure below shows the Shuttle to the same scale as the LRO image.



The Apollo-11 Landing Area I

This photograph was taken from the Lunar Module and it includes both the landing site (arrow) and the Command Module (upper right of arrow). The sharp-rimmed crater at the lower margin is Moltke. The craters north and west of the landing site are secondary craters resulting from ejecta thrown out of Sabine Crater. (NASA photograph AS11-37-5447.)



Problem 1 - From the clues in the photo, what is the scale of this image in meters/mm, and the dimensions of the field-of-view in kilometers?

Problem 2 - What is the smallest feature you can see and its approximate size in meters?

Problem 3 - Identify the Apollo 11 Command Module in the photograph. A) Why can't you use the image scale you calculated in Problem 1 to estimate the size of the Command Module? B) If the moon's surface were 200 km distant, and the Command Module were 1 km distant, can you discover a way to estimate the size of the Command Module from this photo?

Answer Key

Problem 1 - From the clues in the photo, what is the scale of this image in kilometers/mm, and the dimensions of the field-of-view in kilometers?

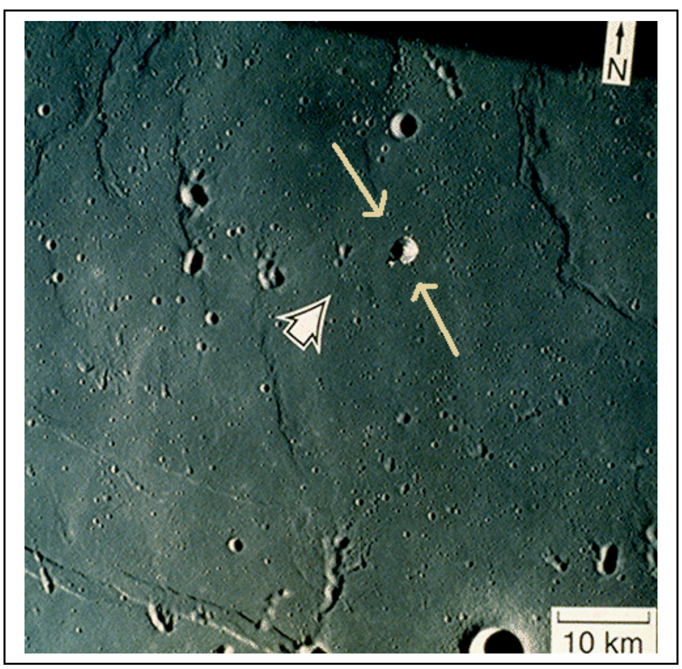
Answer: The legend in the lower-right gives a visual clue about the scale of the image. Place a millimeter ruler across the indicated line to determine its length, about 23 millimeters, so that the scale of the image is then 10 km/23 mm = 431 meters/mm.

Problem 2 - What is the smallest feature you can see and its approximate size in kilometers?

Answer: the smallest craters are about 0.5 millimeters across ,which equals 0.5 x 431 meters/mm = 215 meters.

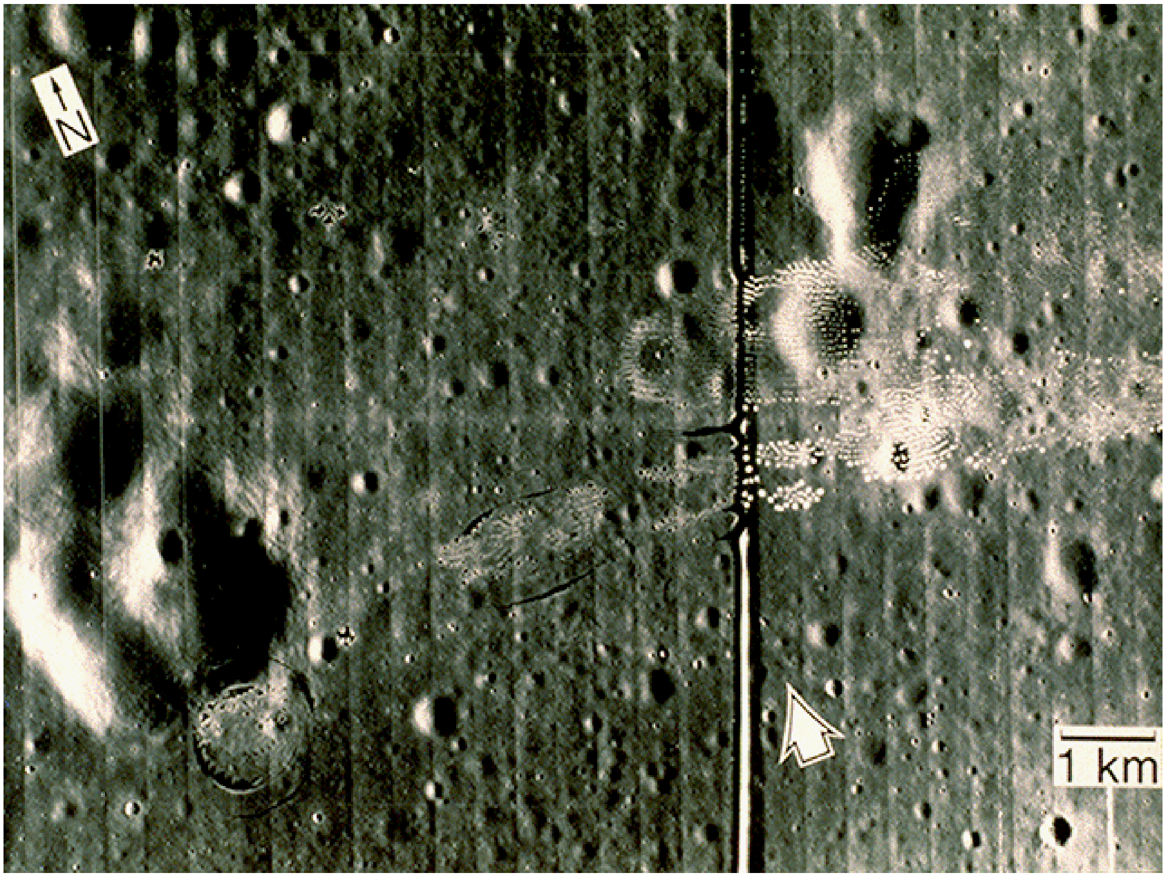
Problem 3 - Identify the Apollo 11 Command Module in the photograph. A) Why can't you use the image scale you calculated in Problem 1 to estimate the size of the Command Module? B) If the moon's surface were 50 km distant, and the Command Module were 1 km distant, can you discover a way to estimate the size of the Command Module from this photo?

Answer: See the object between the two arrows in the photo below. A) You cannot use the 431 meters/mm scale to measure the diameter of the Command Module because the lunar surface, for which the scale applies, is about 200 kilometers further away than the CM. B) You can find a new scaling relationship using similar triangles. 1 millimeter on the lunar surface subtends the same angle as 1 millimeter on the command module. If the lunar size is 431 meters for this angle at a distance of 50 km, then at the Command Module, the same angle will subtend 431 x (1 km / 200 km) = 2.1 meters. That means at the distance of the Command Module, the scale would be 2.1 meters/mm. The Command Module is 6 millimeters across, which corresponds to about 6 x 2.1 = 12.6 meters. The actual size is about 11 meters, so this method works pretty well if you know the exact distances involved, rather than the estimates that were used in the problem.



The Apollo-11 Landing Area - II

This view was obtained by the unmanned Lunar Orbiter V spacecraft in 1967 prior to the Apollo missions to the Moon. The regularly spaced vertical lines are the result of combining individually digitized 'framelets' to make a composite photograph. The irregularly-shaped bright and dark spots are due to film development. The arrow points to the Apollo-11 landing site.



Problem 1 - From the clues in this photograph, what is the scale of the image in meters/mm?

Problem 2 - What is the size of the image field-of-view in kilometers?

Problem 3 - What is the size, in meters, of the largest and smallest features you can identify?

Problem 4 - What is the distance between the tip of the arrow and the large crater?

Problem 5 - By identifying common features, draw a box in the image in 'Lunar11' that is of the same size and orientation as the above image.

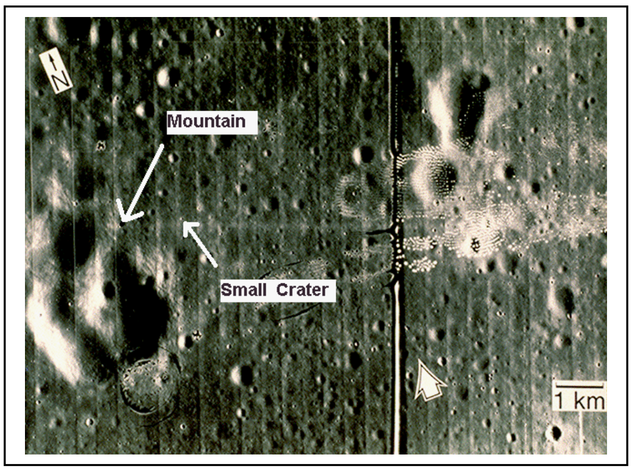
Problem 1 - Answer: The legend in the lower right shows that 1 kilometer equals a length of about 12 millimeters, so the image scale is 1000 meters/12mm = 83 meters/mm.

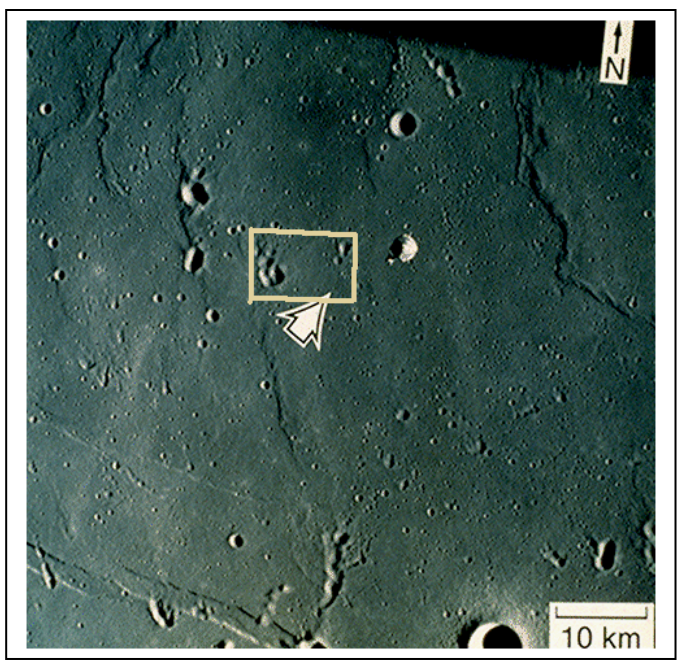
Problem 2 - Answer: The image measures 139 mm wide x 115 mm tall, which equals a physical size of 139 x 83 m/mm = 11.5 km by 115 mm x 83 m/mm = 9.5 km.

Problem 3 - Answer: The large feature in the lower left of the image (below left) is a mountain that measures about 40 mm across or 3.3 km. The smallest crater that is easily recognizable is about 0.5 mm or 41 meters across.

Problem 4 - Answer: This is about 5mm to the edge of the crater or 83 x 5 = 415 meters.

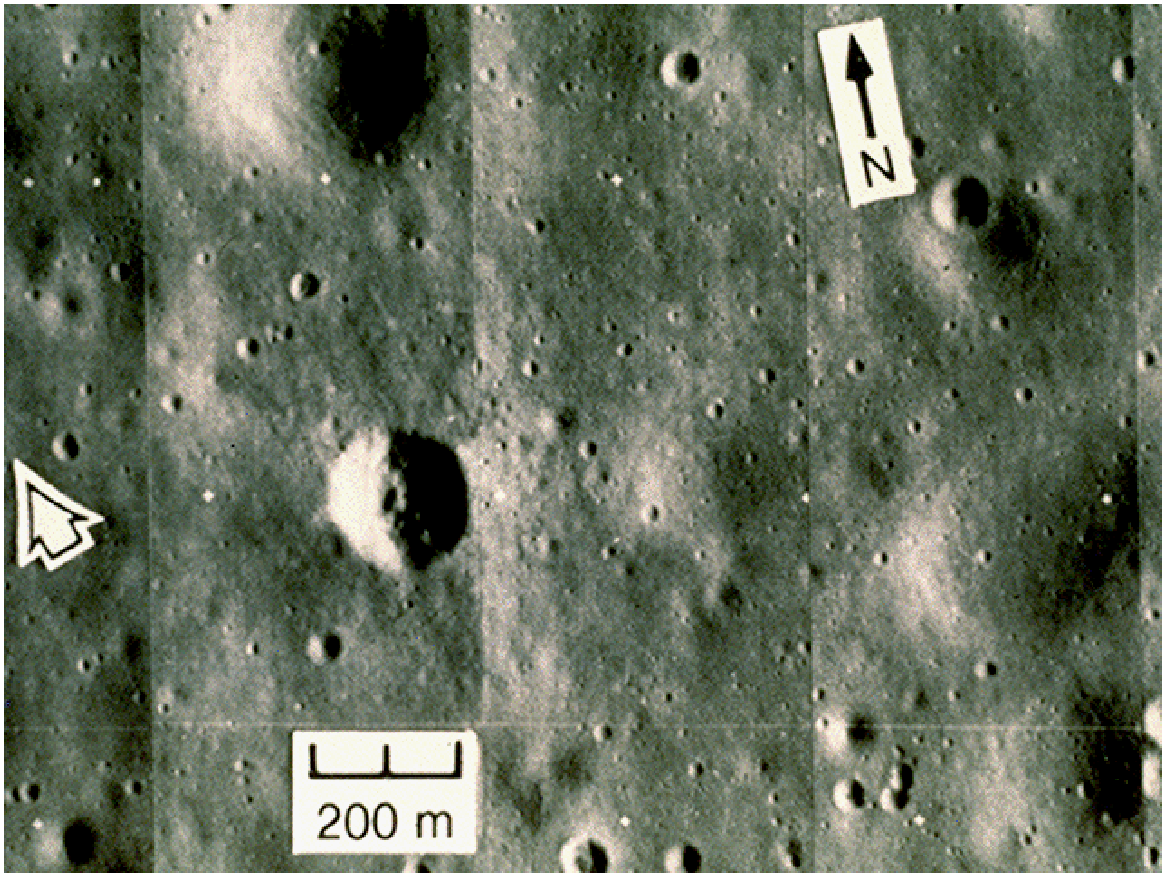
Problem 5 - Answer: In the 'Lunar 11' problem, the scale of that image was 431 meters/mm. The current image has a size of 11.5km x 9.5 km. Since 11.5/0.431 = 26.7 mm and 9.5 km/0.431 = 22.0 mm, the box dimensions are 26.7 x 22.0 mm. The location is approximately shown in the image (below right).





The Apollo-11 Landing Area - III

This photograph was taken in 1967 by NASA's Lunar Orbiter spacecraft. It shows an enlarged view of the Apollo-11 landing area. The fresh crater just to the left of the center is West Crater. The landing site is about 60 meters west of Little West Crater which is located just to the right of the arrow. Astronaut Neil Armstrong visited the rim of Little West Crater while astronaut Edwin Aldrin worked around the Lunar Module.



Problem 1 - What is the scale of this image in meters/mm?

Problem 2 - What is the field of view of this image in kilometers?

Problem 3 - What is the diameter of West Crater in meters?

Problem 4 - What is the diameter of the smallest crater in the field of view?

Problem 5 - If the image had a size of 640 pixels x 480 pixels, what is the resolution of the image in meters/pixel?

Problem 6 - The Apollo-11 Landing Module is about 5 meters across. Could you have spotted this object in the image above if the Lunar Orbiter had taken this picture after 1969?

Problem 7 - Can you locate the Orbiter image within the image shown in Lunar12?

Answer Key

Problem 1 - What is the scale of this image in meters/mm? Answer: The legend mark '200 m' measures 20 mm wide, so the scale is 200 m/20mm = 10 m/mm.

Problem 2 - What is the field of view of this image in kilometers? Answer: The image is 153mm x 105 mm and with the scale of 10 m/mm represents a field 1.5 km x 1.0 km in size.

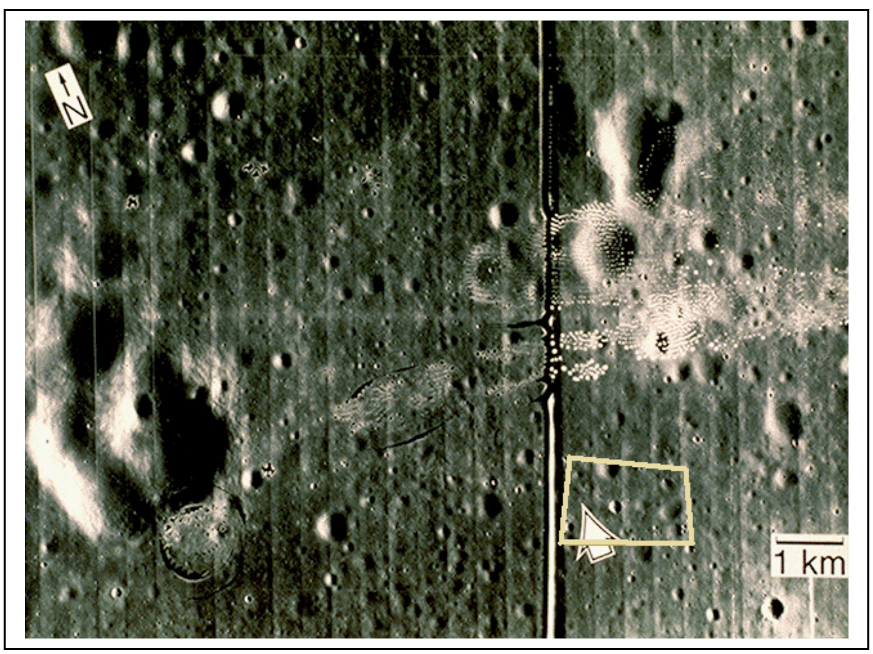
Problem 3 - What is the diameter of West Crater in meters? Answer: The crater measures 18 mm across, which equals 180 meters.

Problem 4 - What is the diameter of the smallest crater in the field of view? Answer: The smallest features are about 1 mm across or 10 meters.

Problem 5 - If the image had a size of 640 pixels x 480 pixels, what is the resolution of the image in meters/pixel? Answer: The width of the field is 1500 meters ,which corresponds to 640 pixels, so the resolution in a digital photograph would be about 1500 meters /640 pixels = 2.3 meters/pixel.

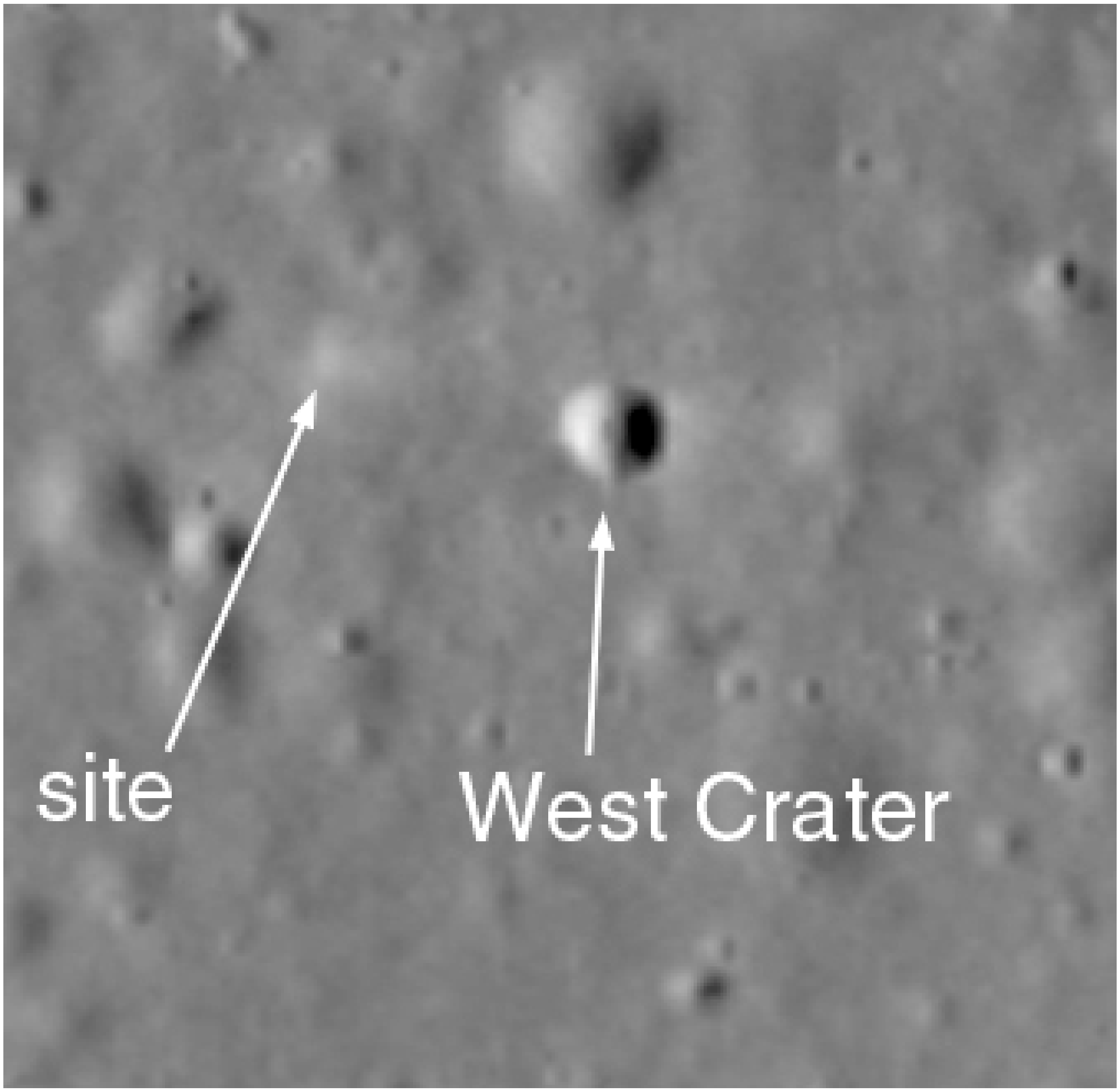
Problem 6 - The Apollo-11 Landing Module is about 5 meters across. Could you have spotted this object in the image above if the Lunar Orbiter had taken this picture after 1969? Answer: On the Orbiter photo, the Lander would measure 5 m / (10m/mm) = 0.5 millimeters across, and would be slightly smaller than the smallest feature that can be seen in the photograph. This also corresponds to about 2 pixels in an equivalent digital image.

Problem 7 - Can you locate the Orbiter image within the image shown in Lunar12?
Answer: See below image for an approximate location.



The Apollo-11 Landing Area - IV

The Apollo 11 landing site was imaged by the Japanese SELENE (Kaguya) satellite on March 12, 2008. The arrow in the photograph below points to lunar soil that was disrupted by the descent rocket plume of the Lunar Landing Module in 1969. It shows that the lighter-colored lunar soil is still exposed 40 years later! In 2009, the Lunar Reconnaissance Orbiter (LRO) will take a series of high-resolution images of the lunar surface in order to identify smooth landing sites for future NASA exploration programs. *(Image courtesy: JAXA/Kaguya)*



Problem 1 - The diameter of West Crater is 180 meters. A) What is the scale of this image in meters/mm? B) What is the size of this field in kilometers?

Problem 2 - By what magnification is this image enhanced over the image in the problem Lunar11?

Problem 3 - During the last minute of the descent, Aldrin and Armstrong worked frantically to avoid landing on large hazards. Select 10 hazards near the landing site and determine their distances at their closest edges, from the touch-down spot.

Answer Key

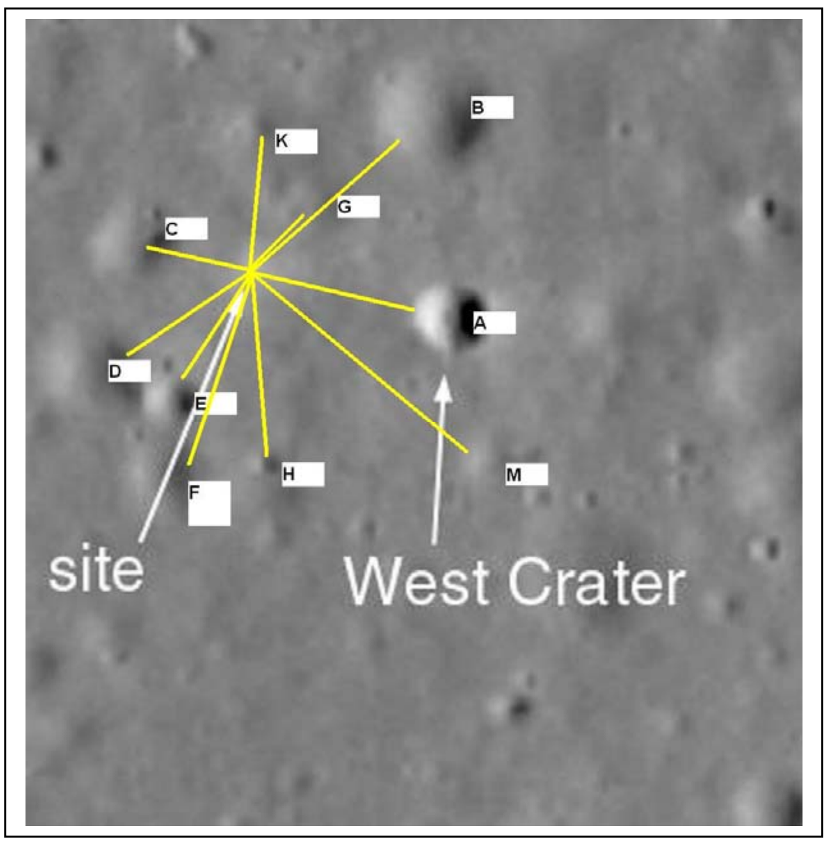
Problem 1 - The diameter of West Crater is 180 meters. A) What is the scale of this image in meters/mm? Answer: The diameter of West Crater is about 14 mm, so the scale of the image is 180 m/14mm = 12.8 meters/mm. B) What is the size of this field in kilometers? Answer: The field is 153 mm wide by 149 mm tall, so at a scale of 12.8 meters/mm, this corresponds to 1.9 km x 1.9 kilometers.

Problem 2 - By what magnification is this image enhanced over the image in the problem Lunar11? Answer: The image in problem Lunar11 had a scale of 431 meters/mm, so the magnification required to get to the scale of the current image is 431/12.8 = 34-times.

Problem 3 - During the last minute of the descent, Aldrin and Armstrong worked frantically to avoid landing on large hazards. Select 10 hazards near the landing site and determine their distances at their closest edges, from the touch-down spot.

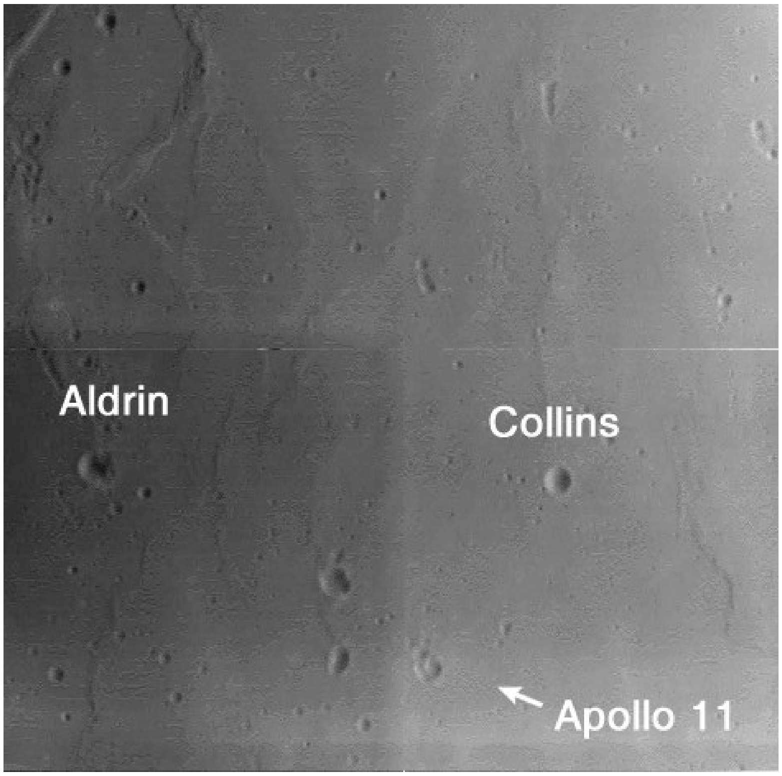
Answer: The table and image below show some possibilities. Note: It may be a good extension exercise to compare these distances with familiar distances in the student's neighborhood or in the school. For example, how large would your schoolyard appear on this photograph?

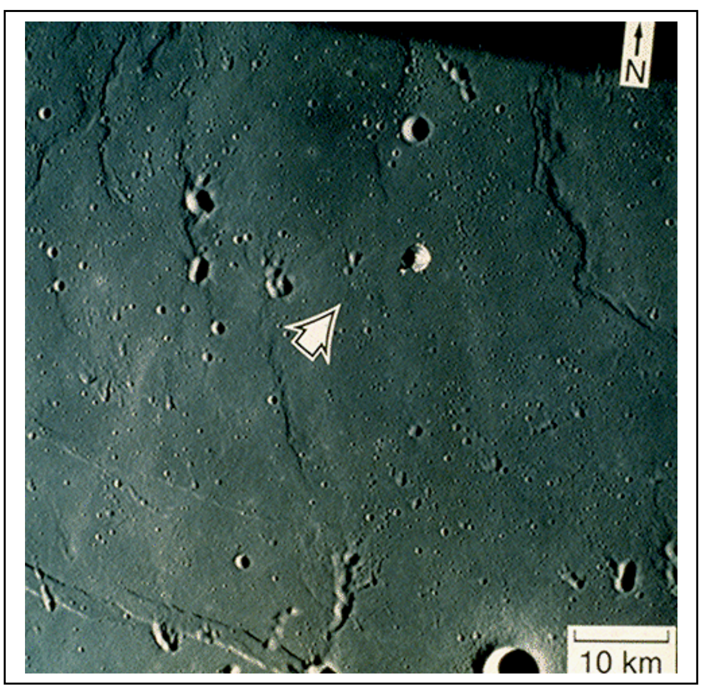
Feature	Distance on image (mm)	Actual Distance (meters)
A	32	410
B	35	448
C	15	192
D	30	384
E	30	384
F	45	576
G	10	128
H	35	448
K	20	26
M	50	640



The Apollo-11 Landing Area - V

This image, taken by the Advanced Moon Imaging Experiment (AMIE) on board ESA's Small Missions for Advanced Research in Technology (SMART-1) spacecraft, shows the Apollo-11 landing site in the Mare Tranquillitatis on the Moon. AMIE obtained the image on 5 February 2006 from a distance of 1,764 kilometers from the surface. The arrow shows the landing site of Apollo-11. The two prominent craters nearby are named after two of the Apollo-11 astronauts. The first man on the Moon, Armstrong, has a crater named after him outside the field of this image.





Problem 1 - The lower figure shows the landing region viewed in 1969 by the Apollo-11 astronauts. Find five features shared by the SMART-1 and Apollo-11 images.

Problem 2 - What is the scale of the lower photograph in meters/mm?

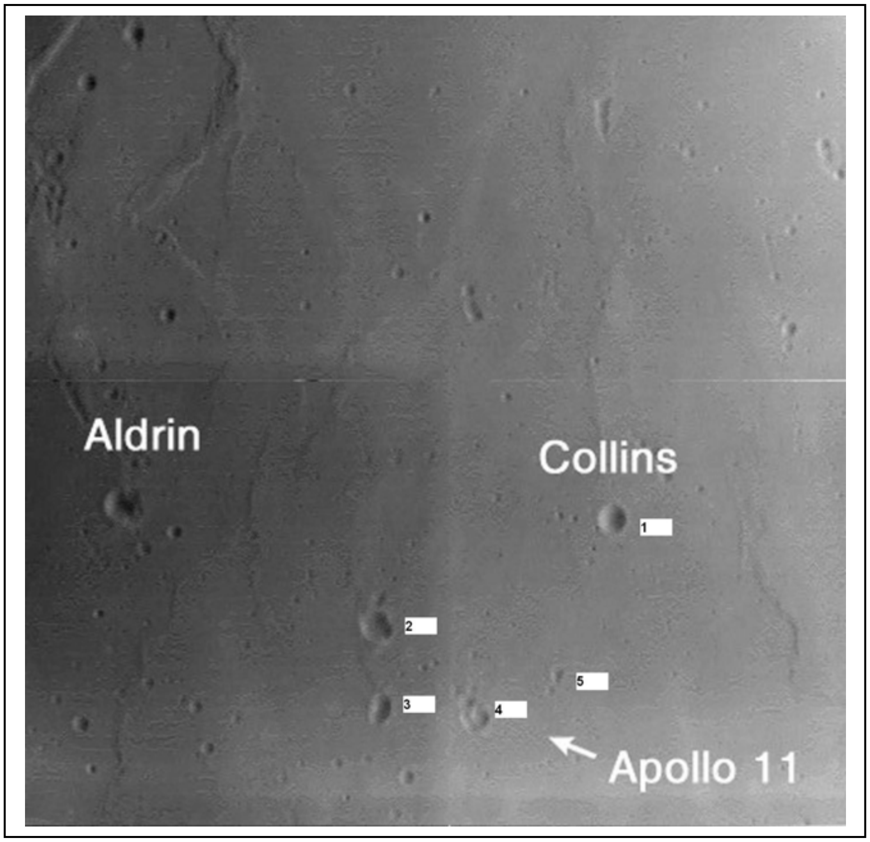
Problem 3 - From your answers to Problem 1 and 2, what is the scale of the SMART-1 image?

Problem 4 - What is the field-of-view of the SMART-1 image, and what is the smallest feature that can be resolved?

Problem 5 - In order to find a safe landing area, astronauts need to be able to see details about 1-meter across. Using the same camera as SMART-1, how much closer to the moon would the SMART-1 satellite have to be to see details at this scale?

Answer Key

Problem 1 - The lower figure shows the landing region viewed in 1969 by the Apollo-11 astronauts. Find five features shared by the SMART-1 and Apollo-11 images. Answer: See the figure below for clues!

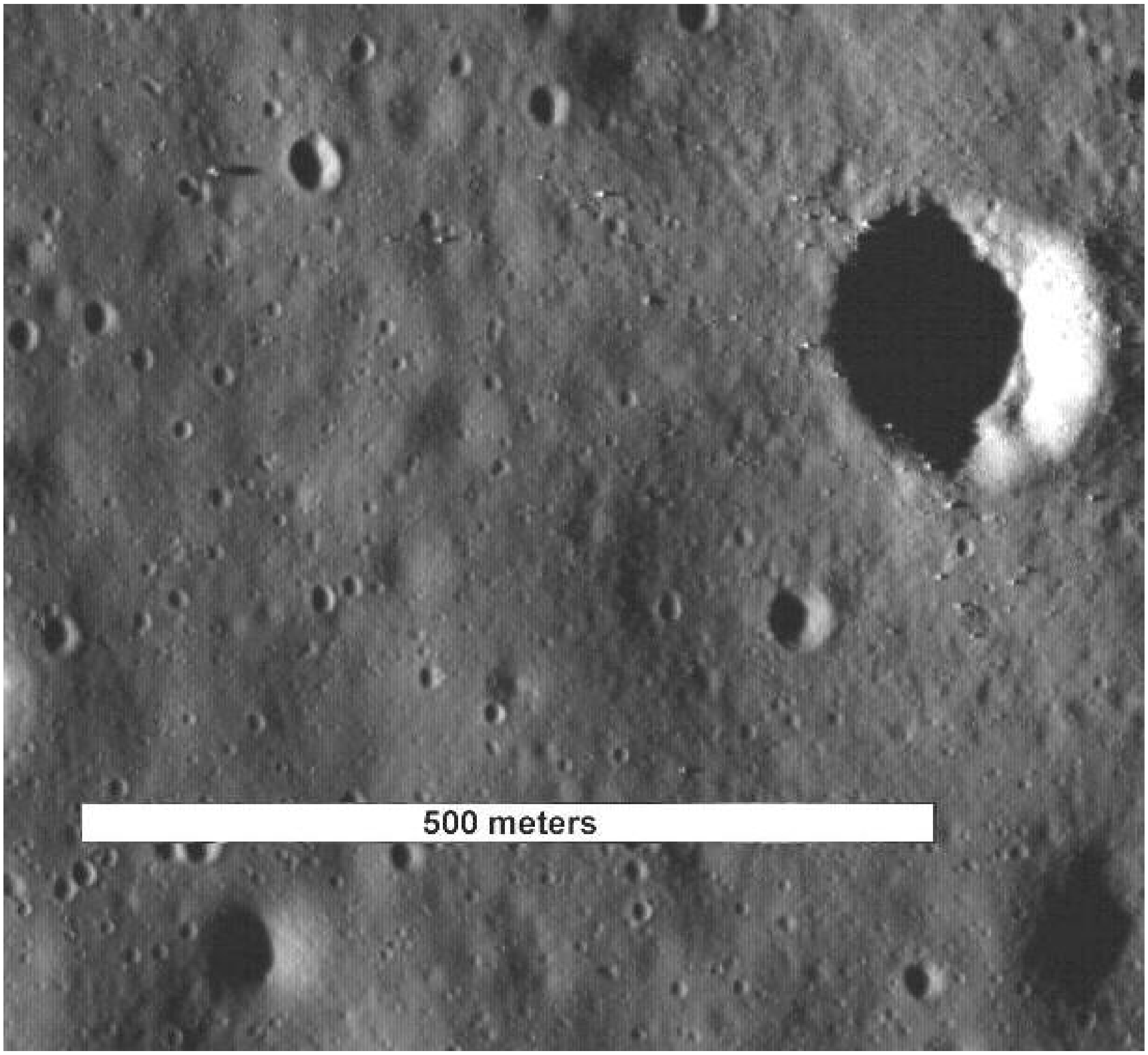


Problem 2 - What is the scale of the lower photograph in meters/mm? Answer: Measuring the Legend indicates that 10 km = 12 mm so the scale is 833 meters/mm.

Problem 3 - From your answers to Problem 1 and 2, what is the scale of the SMART-1 image? Answer: Take features 1 and 2 and measure their center-to-center separations on the Apollo-11 image. From the scale of 833 m/mm, they are separated by 30mm x 833 = 25 kilometers. On the SMART-1 photo, these two features are 33 mm apart, so the scale is 25 km/33 mm = 757 meters/mm.

Problem 4 - What is the field-of-view of the SMART-1 image, and what is the smallest feature that can be resolved? Answer: 103 mm x 101 mm at a scale of 757 m/mm becomes 78 km x 76 km.

Problem 5 - In order to find a safe landing area, astronauts need to be able to see details about 1-meter across. Using the same camera as SMART-1, how much closer to the moon would the SMART-1 satellite have to be to see details at this scale? Answer: The SMART-1 satellite was at 1,764 km for a scale of 767 meters/mm. To get 1 meter/mm it needs to be 767-times closer or 2.3 kilometers from the surface. Note, by using a camera lens that is 30x, it need only be at an altitude of 69 km!



The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows a region near the Apollo-11 landing site. The Lunar Module (LM) can be seen from its very long shadow near the large crater in the upper left corner of the image.

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Problem 2 - Measure all of the craters larger than or equal to 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$: the Areal Crater Density (craters/km^2).

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Answer: The 500-meter bar is 111 millimeters long so the scale is 500 M/111mm = **4.5 meters/mm. The image has the dimensions of 149 mm x 136 mm or 670m x 612 m for an area of 0.41 kilometers^2.**

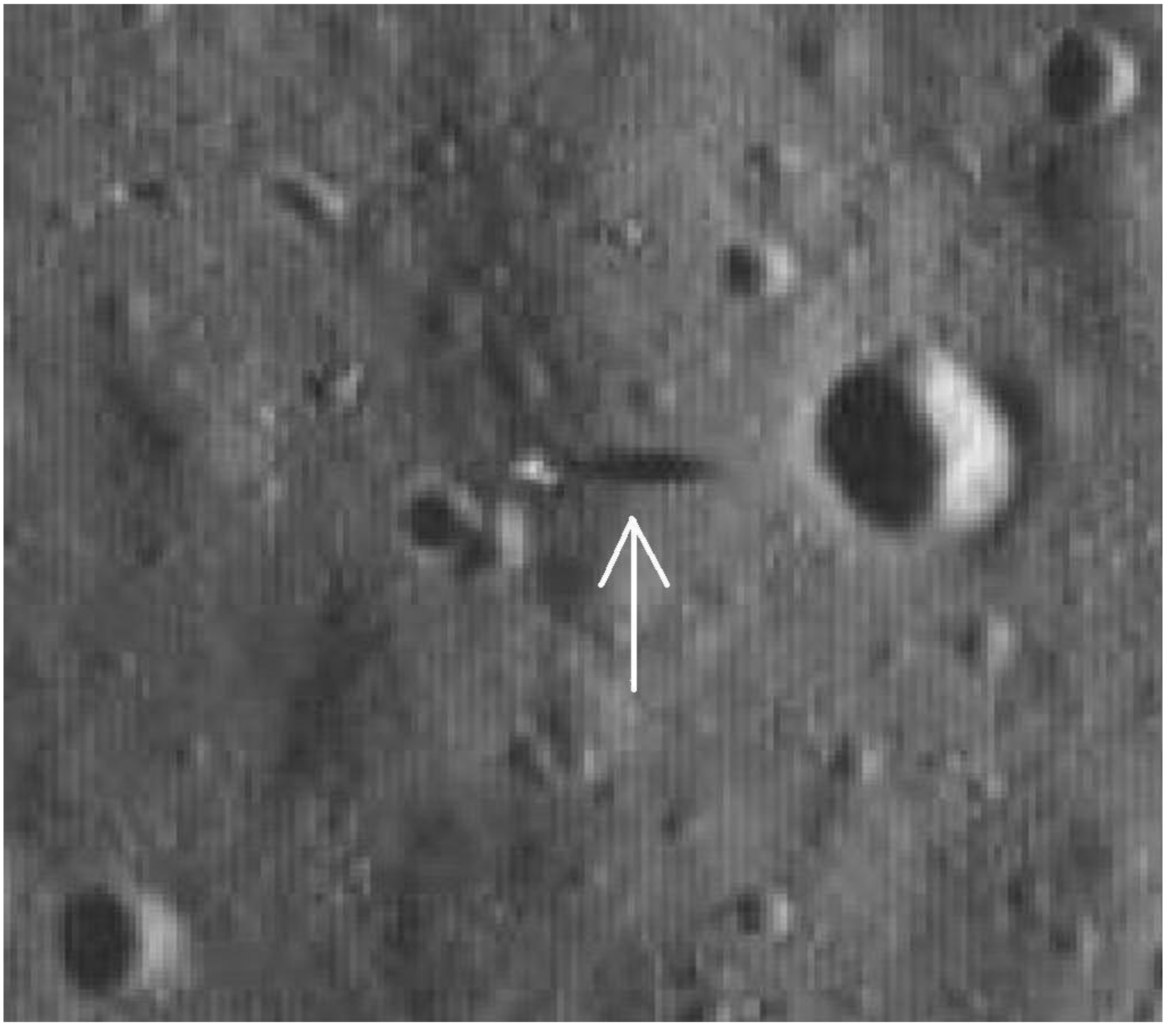
Problem 2 - Measure all of the craters larger than 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$: the Areal Crater Density (craters/km^2). Answer: The following table shows an example. Students bin intervals may differ.

Crater diameter (mm)	Crater diameter (meters)	Number of craters close to this size	Areal Density	**Problem 3** Average distance in kilometers
2 mm	9	70	70/0.41 = 171	**0.08**
4 mm	18	6	15	**0.25**
6 mm	27	3	7	**0.38**
8 mm	36	2	5	**0.44**
10 mm	45	1	2	**0.71**

Students may extend this table to include craters of 1-mm diameter and also the single, very large crater that is 35mm in diameter. The number of counted craters, especially in the smallest bins, will vary. Student data may be averaged together to improve accuracy in each bin.

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

Answer: See above table for tabulated values. Students may also convert the answers to meters. For example, '0.08 km' = 80 meters. Students will need to estimate the Areal Crater Density for craters just below the tabulated threshold of 9 meters. This can be done by estimating the shape of the plotted curve through the points, and extrapolating it to 5 meters. It is also possible to use Excel Spreadsheets by entering the data and plotting the 'scatter plot' with a trendline added. Reasonable values for the Areal Creater Density would range from 171 craters/km^2 to 1000 craters/km^2 ,which lead to distances **between 80 meters and 30 meters, but probably closer to 30 meters given the rapidly decreasing trend of the curve based on the data in the bins for 18-meter and 9-meter crater diameters**



The LRO satellite recently imaged the Apollo 11 landing area on the surface of the moon. The above (172 pixels wide x 171 pixels high) image shows this area and is 172 meters wide.

Problem 1 - Determine the scale of the image in meters per millimeter and meters per pixel? What is the diameter, in meters, of A) the largest crater? B) the smallest crater?

Problem 2 - The shadow identified by the arrow was cast by the Lunar Landing Module which is about 3.5 meters tall. Using A) trigonometry, or a B) scaled drawing and a protractor, what was the sun angle at the time of the photograph?

Problem 3 - Are there any individual boulders larger than 1 meter across in this area?

Problem 1 - Determine the scale of the image in meters per millimeter and meters per pixel? What is the diameter, in meters, of A) the largest crater? B) the smallest crater?

Answer: The image is 153 millimeters wide, which corresponds to 172 meters, so the scale is **1.1 meters per millimeter,** and the image is 172 pixels wide so the resolution is 172 pixels/153 meters = **1.1 meters/pixel.**

The largest crater is about 25mm x 30 mm in size, which corresponds to 25mm x 1.1 meters/mm = 28 meters wide, and 30 mm x 1.1 = 33 meters long, for an **average size of about 30 meters across.** B) The smallest discernable features are about 1 to 2 mm wide, which corresponds to an actual size of about 1-2 pixels or 1 to 2 meters. Note, there can be no actual details smaller than the pixel resolution of the image (1.1 meters).

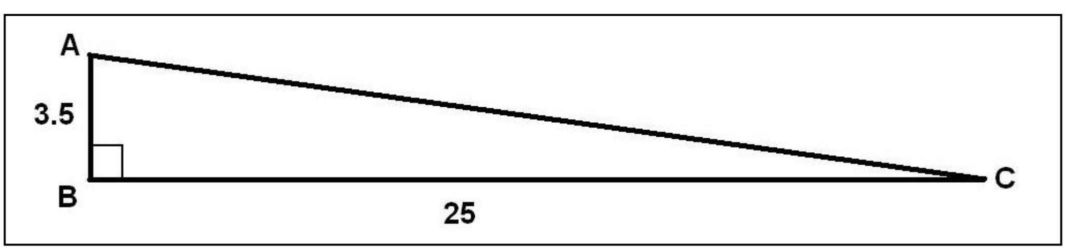
Problem 2 - The shadow near the center of the picture was cast by the Lunar Landing Module which is about 3.5 meters tall. Using A) trigonometry, or a B) scaled drawing and a protractor, what was the sun angle at the time of the photograph?

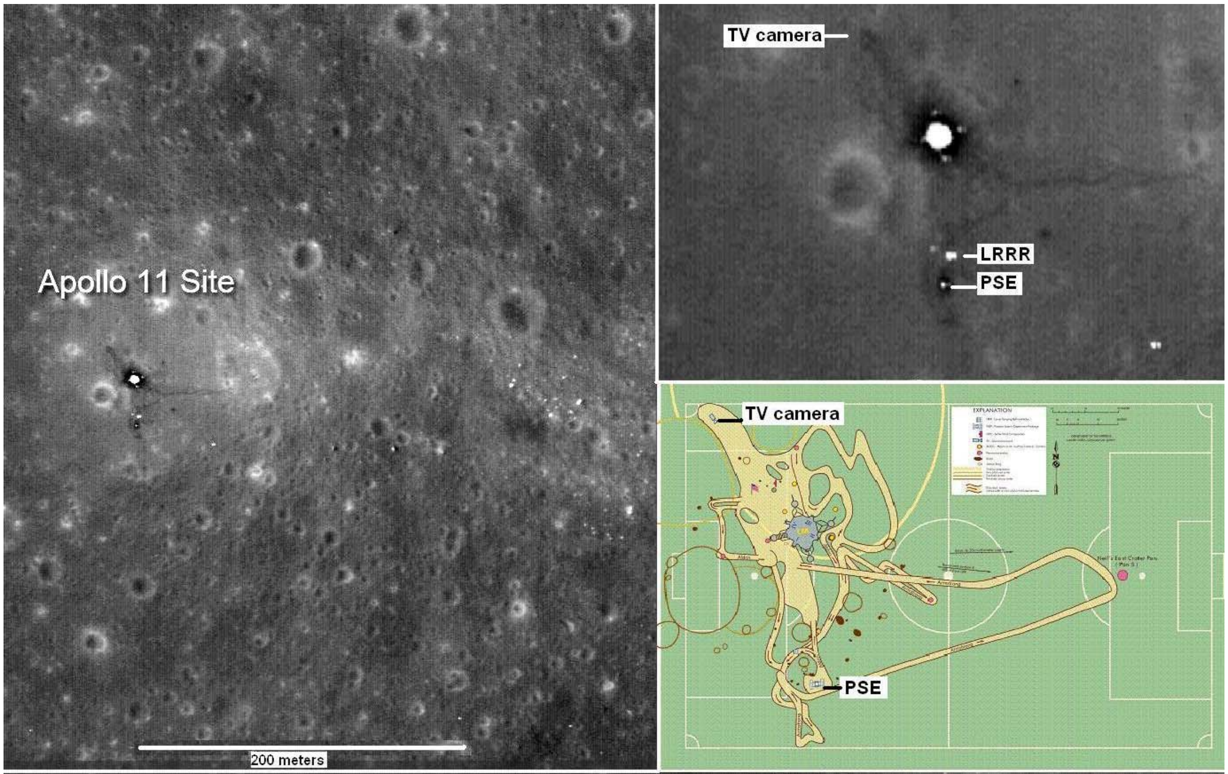
Answer: The length of the shadow from the base of the lander is about 23 millimeters or in actual length, 23 x 1.1 = 25 meters. This makes a right triangle, ABC, with a base length AB= 25 meters and an altitude of AC=3.5 meters and a hypotenuse located along BC, with the right-angle defined as ABC.

Method 1: From trigonometry, Tan(theta) = 3.5 meters/25 meters = 0.14 so the angle whose tangent is 0.14 is **theta=8.0 degrees.**

Method 2: A scaled drawing is shown below, and a protractor may be used to measure the angle directly from the diagram.

Problem 3 - Are there any individual boulders larger than 1 meter across in this area?
Answer: No, because they would have shadows about 7 meters long (1/4 the Apollo 11 module) and there are no such shadows in the image, other than the Apollo-11 Landing Module itself. This area of the moon seems to be boulder-free at a resolution of 1 meter, which is why it was selected by Apollo-11 astronauts for a landing site.





The Lunar Reconnaissance Orbiter (LRO) recently imaged the Apollo-11 landing area at high-resolution and obtained the image above (Top left). An enlargement of the area is shown in the inset (Top right) and a rough map of the area is also shown (bottom right). The landing pad with three of its four foot-pads is clearly seen, together with the Lunar Ranging Retro Reflector experiment (LRRR), the Passive Seismic Experiment (PSE) and the TV camera area. The additional white spots seen in the left image are boulders from the West Crater located just off the right edge of the image.

Problem 1 - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?

Problem 2 - About what is the distance between the TV camera and the PSE?

Problem 3 - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?

Problem 4 - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater?

Problem 5 - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?

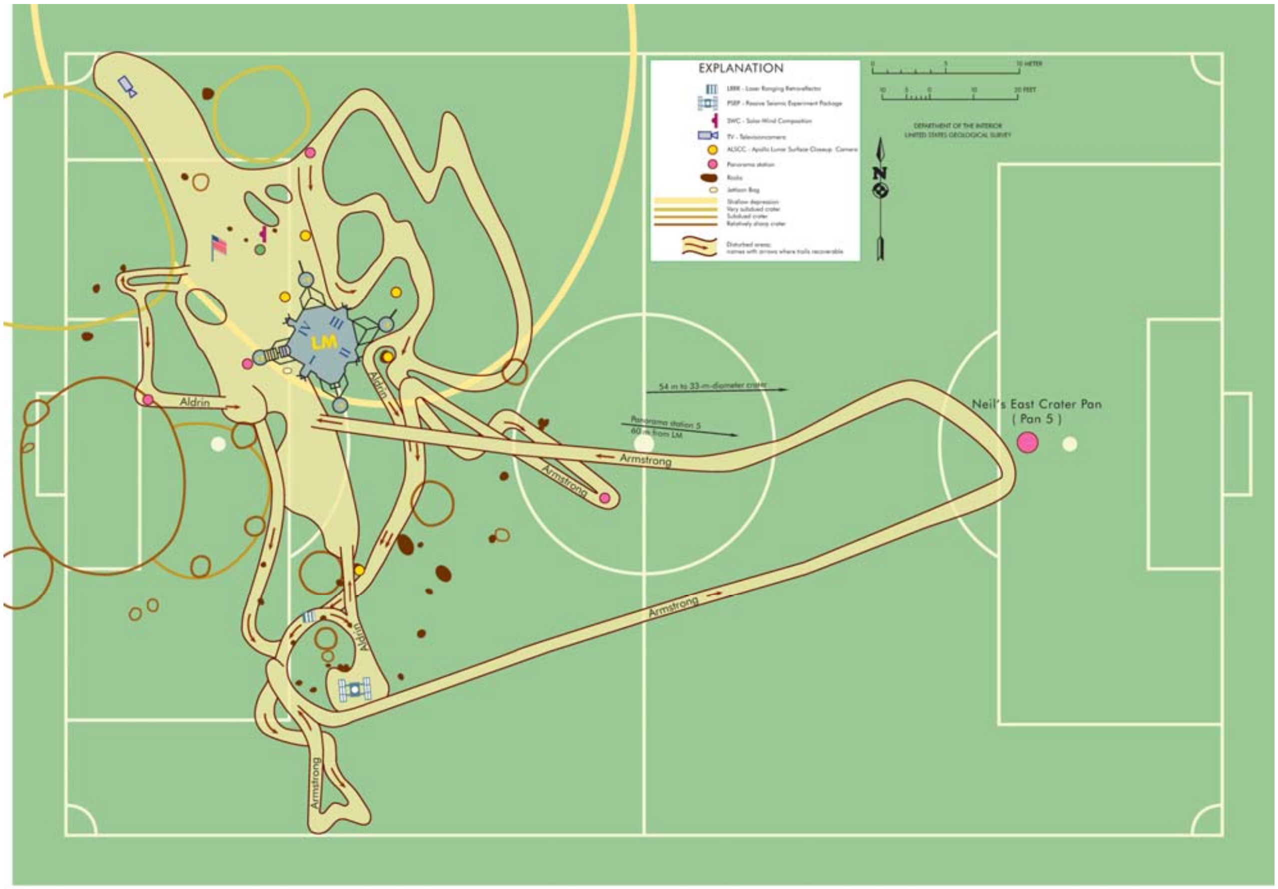
Problem 1 - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?
Answer: On the main image, the length is 43 millimeters so the scale is 200 meters/43 mm = **4.7 meters/mm for the left-hand image.** The distance between the landing pad and the LRRR on this image is 5 millimeters or 5 x 4.7 = 24 meters. In the upper right image, the landing pad and the LRRR are 16 mm apart, so the scale of this image is 24 meters/16mm = **1.5 meters/mm**. The PSE and landing pad are clearly indicated in the map, which the top image says are 20 mm or 20x1.5 = 30 meters apart. On the map, these points are also 20 mm apart, so the scale is also 1.5 meters/mm on the map.

Problem 2 - About what is the distance between the TV camera and the PSE?
Answer: According to the map, the distance is 38 millimeters or 38 mm x (1.5 m/mm) = **57 meters apart**.

Problem 3 - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?
Answer; A) Height x Width = 101mm x 86mm and for a scale of 4.7 meters/mm this equals **475m x 404m**. B) The are in square kilometers is 0.475 m x 0.404 m= **0.19 km^2.**

Problem 4 - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater? Answer: A) The largest circular feature is about 8mm in diameter or **38 meters across**. B) The smallest feature is about 1 millimeter across or **4.7 meters**.

Problem 5 - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?
Answer: Depending on the quality of the printed copy, students may count between 20 and 100 craters. Assuming the lower value, the crater density is 20 craters/0.19 km^2 = 105 craters/km^2. If the PDF file is displayed on the computer screen, a much better contrast is obtained and students should be able to count about 225 craters for a density of 1,200 craters/km^2. **Values between 100 and 1000 craters/km^2 are acceptable**.



The NASA Lunar Reconnaissance Orbiter (LRO), launched in 2009, will be able to see details on the lunar surface at much higher resolution than any previous lunar mapping mission. The images will have a resolution of about 0.7 meters per pixel. The map shows the area surrounding the Apollo 11 landing site where astronauts deployed experiments and walked around the landing site to gather rock and soil samples. The grid lines show a standard soccer field in comparison, with a distance between the white goal boundaries of 110 meters.

Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels.

Problem 3 - Will LRO be able to see: A) the Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

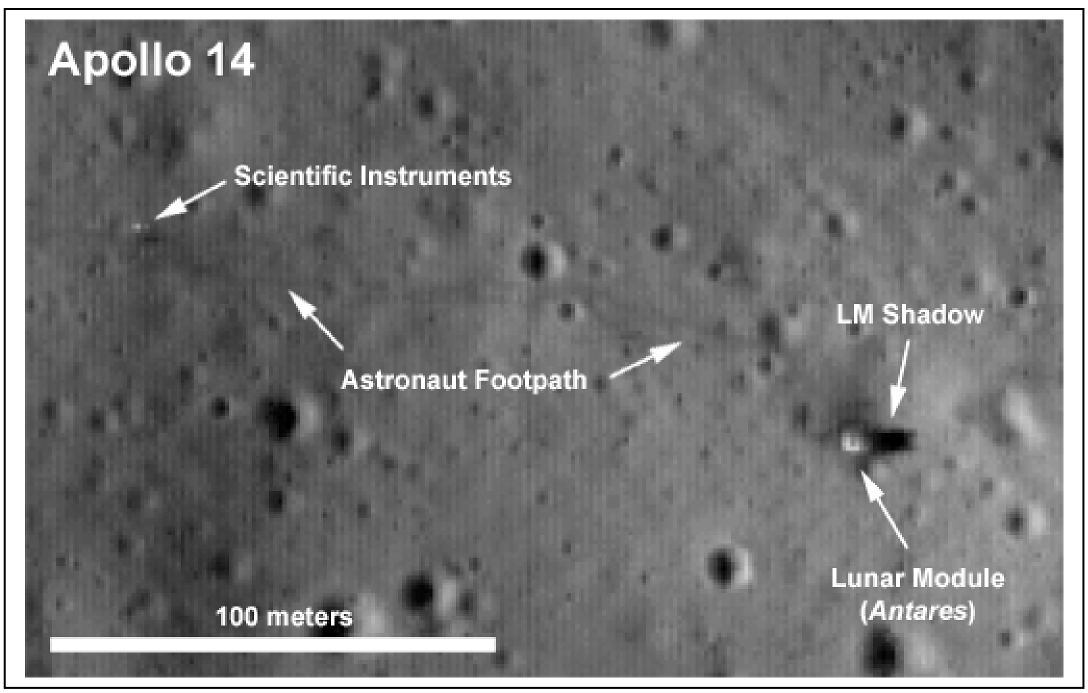
Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

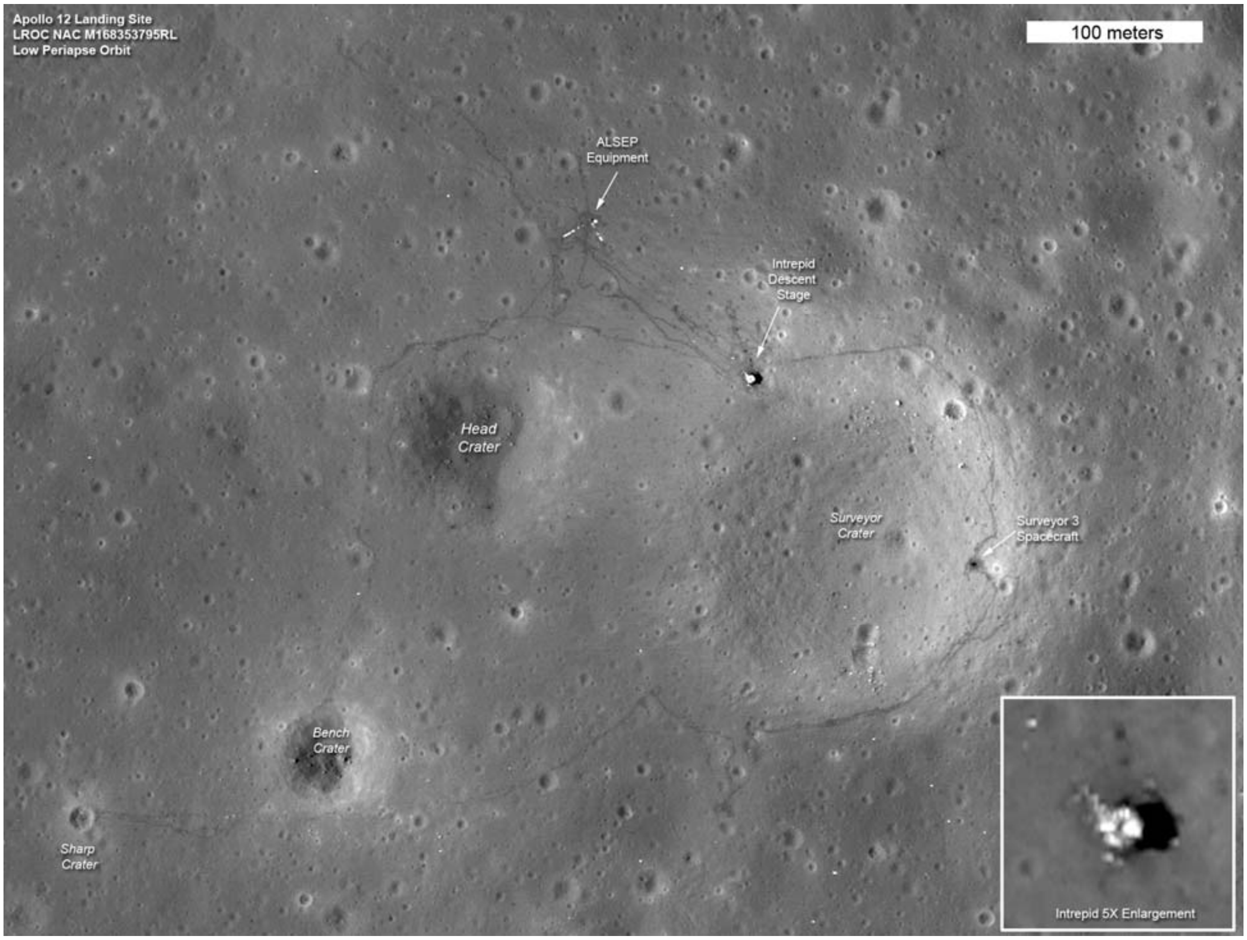
Answer: A millimeter ruler placed along the long-edge of the figure would indicate that the distance between the vertical white goal lines is 153 millimeters .Since this corresponds to 110 meters on the actual field, the scale of this figure is 110 meters/153 mm = 0.7 meters per millimeter. Alternately, students may measure the width of the green area which is 167 millimeters and corresponds to 120 meters.

Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels. Answer: The LRO pixels are 0.7 meters wide, so the students would have to draw a grid with rows and columns 1 millimeter wide.

Problem 3 - Will LRO be able to see: A) The Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

Answer: Below is an example of the details seen by the LRO cameras at the Apollo 14 landing site!





NASA's Lunar Reconnaissance Orbiter (LRO) from a lunar orbit of 21 kilometers (13 miles) captured the sharpest images ever taken from space of the Apollo 12, 14 and 17 landing sites. Images show the twists and turns of the paths made when the astronauts explored the lunar surface. One of the details that shows up is a bright L-shape in the Apollo 12 image. It marks the locations of cables running from ALSEP's central station to two of its instruments. Although the cables are much too small for direct viewing, they show up because they reflect light very well.

Problem 1 – Following one of the walking paths, about how many meters did the astronauts have to walk from A) the ALSEP to the Descent Stage, and then around Surveyor Crater to finally reach the Surveyor spacecraft? B) The Surveyor spacecraft to Sharp Crater?

Problem 2 – Using your favorite method, about how many craters can you see across this entire area?

Problem 3 - If the craters were created over a period of about 3 billion years, about what may have been the average time between impacts to form the craters you see?

Problem 1 – Following one of the walking paths, about how many meters did the astronauts have to walk from A) the ALSEP to the Descent Stage, and then around Surveyor Crater to finally reach the Surveyor spacecraft? B) The Surveyor spacecraft to Sharp Crater?

Answer: Print out the problem on a typical laser printer and measure the '100 meter' bar with a millimeter ruler. An answer of about 23 millimeters yields an image scale of about 100 meters/23 mm = 4.3 meters/mm.

A) Using a piece of string or a millimeter ruler, measure the segments of the thin black 'track' that astronauts took. An answer of about 90 millimeters will be adequate. Using the image scale of 4.3 meters/mm you will get a distance of 90 mm x (4.3 m/mm) = 387 meters. This can be rounded to **390 meters**.

B) Measuring the track segments, a string length of about 200 millimeters is adequate. From the scale factor, this equals a physical distance of 200 x 4.3 = **860 meters** traveled.

Problem 2 – Using your favorite method, about how many craters can you see across this entire area?

Answer: Divide the area into a grid of squares. Count the number of craters you can see in one square, and multiply by the total number of squares. For example, if you make the squares 40mm x 40mm, you can fit 4 columns and 3 rows. Selecting the one in the second column, first row, you can count about 80 craters (from 0.5 to 2 millimeters across on the image) so the total number of craters is about 80 x 12 = 960 craters. **Answers between 800 and 1100 are also reasonable estimates.**

Note: From the image scale, the most common craters range in size from 0.2 millimeters to 2 millimeters, which corresponds to an actual size between 0.9 meters and 8.6 meters.

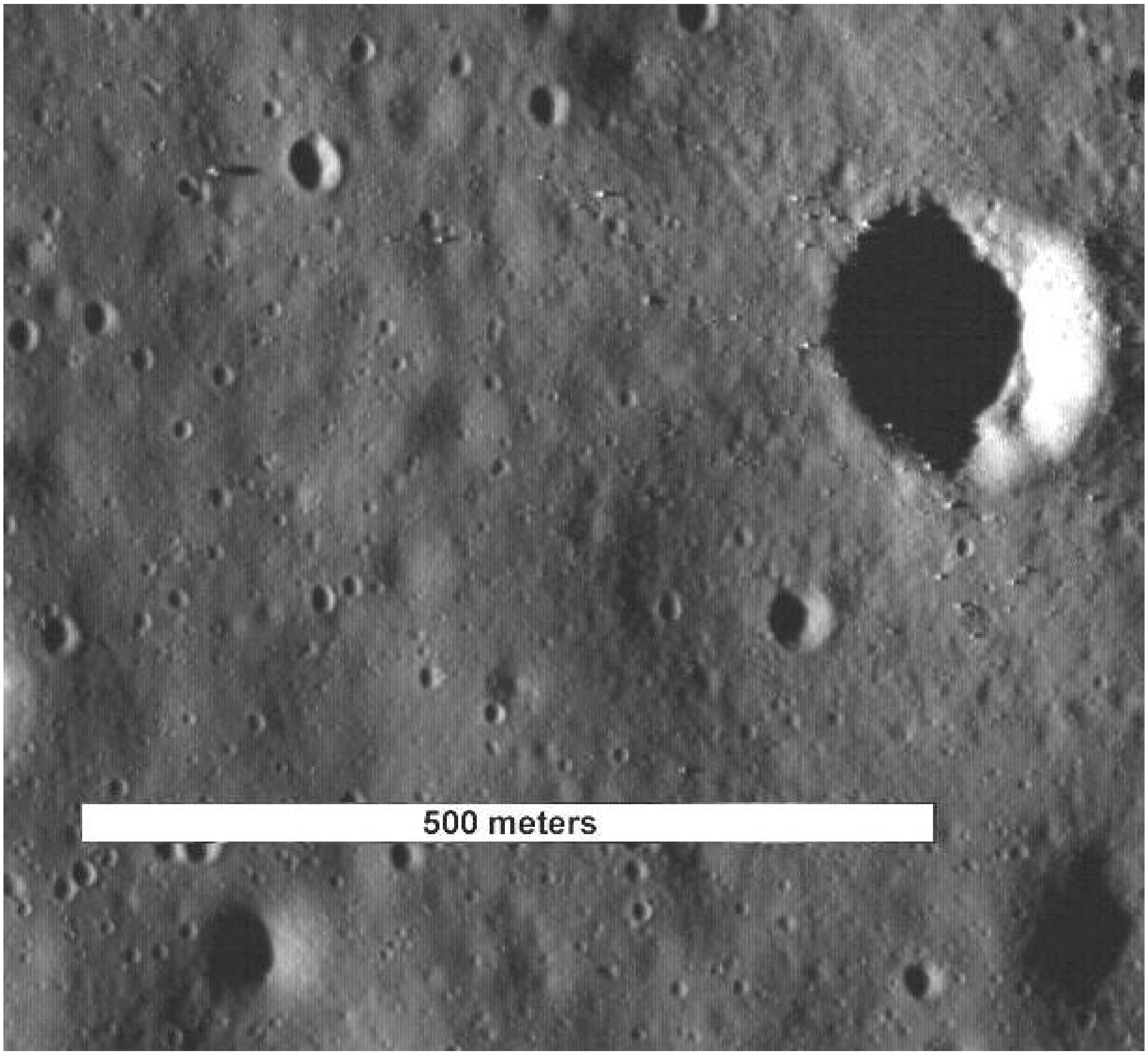
Problem 3 - If the craters were created over a period of about 3 billion years, about what may have been the average time between impacts to form the craters you see?

Answer: If we select 1000 craters as the average estimate, then the rate of cratering is about 1000 craters / 3 billion years or **1 crater every 3 million years**.

For more information about these images, see the NASA press release at:

NASA Spacecraft Images Offer Sharper Views of Apollo Landing Sites
Sep 6, 2011

http://www.nasa.gov/mission_pages/LRO/news/apollo-sites.html



The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows the region near the Apollo-11 landing area. The Lunar Module (LM) and its shadow are shown to the left of the large crater in the upper right corner. Sunlight comes from the left, and so craters will have their shadow zones on the left-hand side of their depressions. Objects above the surface, like the Apollo LM, will be bright on the left side, and have their right-side in shadow.

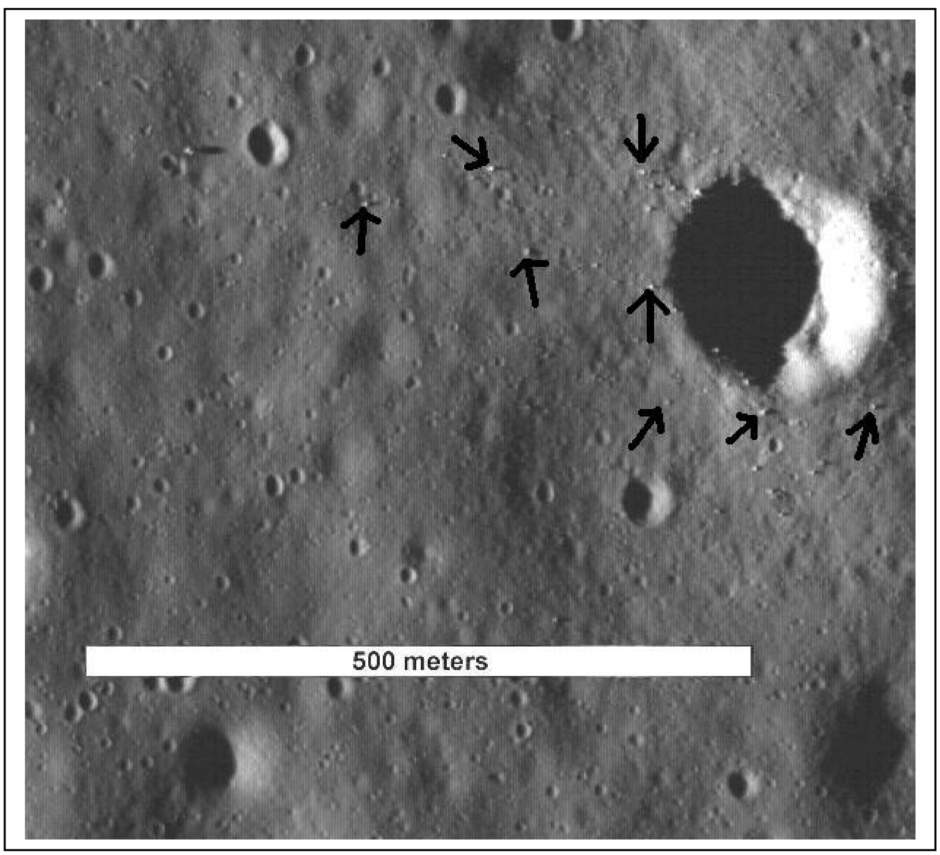
Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B) the length of the Apollo LM shadow.

Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?

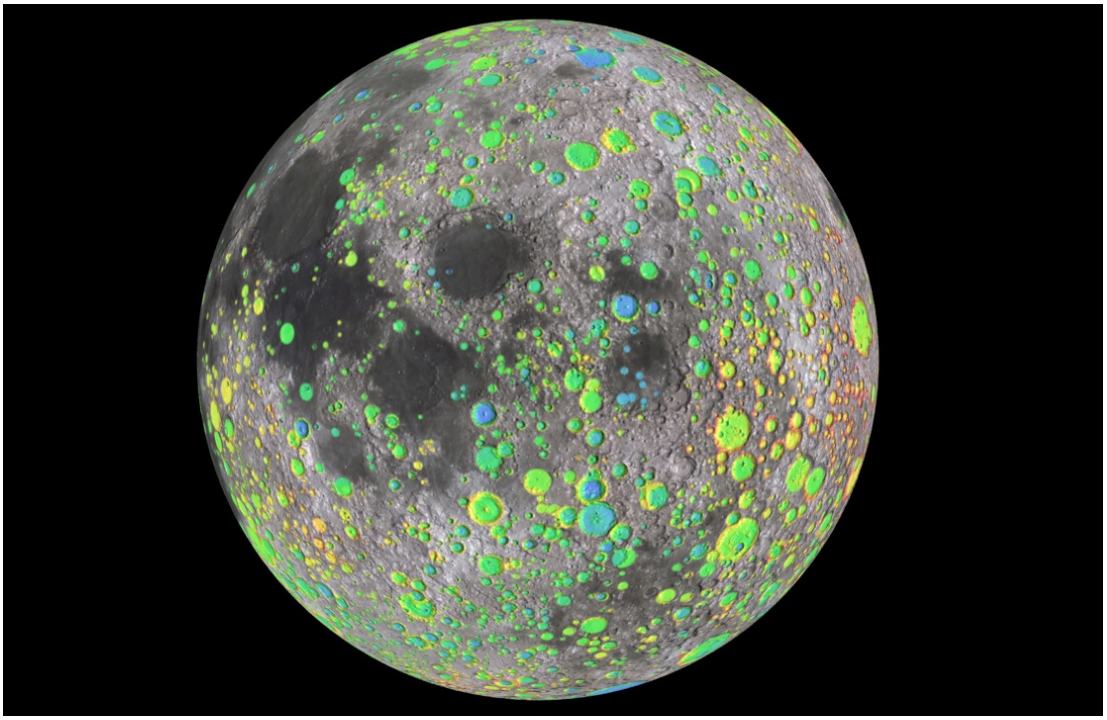
Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B) the length of the Apollo LM shadow.

Answer: Measure the length of the white bar on the image, which corresponds to a physical length of 500 meters on the lunar surface. Students should get answers near 111 millimeters. A) so the scale of the image is 4.5 meters/millimeter. The length of the LM shadow is 6 mm, so its physical length is 6 x 4.5 = 27 meters.

Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?
Answer: The image below shows some examples. The LM is 3.5 meters tall and casts a 27-meter shadow, so by using similar triangles and proportions, that means that a 1-meter boulder will cast a shadow that is 1/3.5 x (27 meters) = 7.7 meters long. At the image scale of 4.5 meters/mm, that corresponds to a length on the image that is just under 2-mm long. Students may create tables for actual, numbered, boulders in the image and determine more accurate boulder sizes. Students should realize that, although they cannot directly see boulders smaller than about 1-pixel (1.4 meters) they can easily see the shadows of boulders much smaller than this. For example, a shadow that is 1 millimeter long is from an unobservable boulder about 0.5 meters across!



Investigating the History of Lunar Cratering



The Lunar Reconnaissance Orbiter used millions of measurements of the lunar surface to establish the history of cratering on the surface.

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.
Answer: The image diameter is 90 millimeters so the scale is 3400 km/90 mm = **38 km/mm.**

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?
Answer: 70 km equals 2 millimeters at this image scale. There are about **56 craters larger than 2 mm on the image. Students answers may vary from 40 to 60.**

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?
Answer: You would expect to find about 0.2 x 56 = **11 craters larger than 70 km.**

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Answer: **The lunar highlands were present first and were impacted by asteroids until just before the maria formed. There are few/no craters in the maria regions larger than 70 km, so the maria formed after the episode of large impactors ended.**

For more information, see the LRO press release at:

"LRO Exposes Moon's Complex, Turbulent Youth"

http://www.nasa.gov/mission_pages/LRO/news/turbulent-youth.html

The image of Mercury's surface on the left was taken by the MESSENGER spacecraft on March 30, 2011 of the region near crater Camoes near Mercury's south pole. In an historic event, the spacecraft became the first artificial satellite of Mercury on March 17, 2011. The image on the right is a similar-sized area of our own Moon near the crater King, photographed by Apollo 16 astronauts.

The Mercury image is 100 km wide and the lunar image is 115 km wide.

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Answer: Mercury image width is 80 mm, scale is 100 km/80mm = **1.3 km/mm**
Moon image width = 68 mm, scale is 115 km/68mm = **1.7 km/mm**

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Answer: Mercury = 0.5 mm x (1.3 km/mm) = **700 meters**.
Moon = 1.0 mm x (1.7 km/mm) = **1,700 meters**.

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Answer: **On Mercury, less of the material ejected by the impact gets away, and so more of it falls back to the surface near the crater. For the Moon, the escape velocity is so low that ejected material can travel great distances, or even into orbit and beyond, so less of it falls back to the surface to make additional craters**.

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Answer: The volume of a sphere is given by $4/3 \pi R^3$, so if you increase the radius of a sphere by a factor of 1.4, you will increase its volume by a factor of 1.4^3 = 2.7 times, so the volume of Mercury is **2.7 times** larger than the volume of the Moon.

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Answer: The density of Mercury is a factor of 5400/3400 = 1.6 times that of the Moon. Since the volume of Mercury is 2.7 times larger than the Moon, the mass of Mercury will be density x volume or 1.6 x 2.7 = **4.3 times than of the Moon**.

Note: Actual masses for Mercury and the Moon are 3.3×10^{23} kg and 7.3×10^{22} kg respectively, so that numerically, Mercury is 4.5 times the Moon's mass...which is close to our average estimate.

Lunar Cratering - Probability and Odds

The moon has lots of craters! If you look carefully at them, you will discover that many overlap each other. Suppose that over a period of 100,000 years, four asteroids struck the lunar surface. What would be the probability that they would strike an already-cratered area, or the lunar mare, where there are few craters?

Problem 1 - Suppose you had a coin where one face was labeled 'C' for cratered and the other labeled U for uncratered. What are all of the possibilities for flipping C and U with four coin flips?

Problem 2 - How many ways can you flip the coin and get only Us?

Problem 3 - How many ways can you flip the coin and get only Cs?

Problem 4 - How many ways can you flip the coin and get 2 Cs and 2 Us?

Problem 5 - Out of all the possible outcomes, what fraction includes only one 'U' as a possibility?

Problem 6 - If the fraction of desired outcomes is 2/16, which reduces to 1/8, we say that the 'odds' for that outcome are 1 chance in 8. What are the odds for the outcome in Problem 4?

A fair coin is defined as a coin whose two sides have equal probability of occurring so that the probability for 'heads' = 1/2 and the probability for tails = 1/2 as well. This means that P(heads) + P(tails) = 1/2 + 1/2 = 1. Suppose a tampered coin had P(heads) = 2/3 and P(tails) = 1/3. We would still have P(heads) + P(tails) = 1, but the probability of the outcomes would be different…and in the cheater's favor. For example, in two coin flips, the outcomes would be HH, HT ,TH and TT but the probabilities for each of these would be HH = (2/3)x(2/3)=4/9; HT and TH = 2 x (2/3)(1/3) = 4/9, and TT = (1/3)x1/3) = 1/9. The probability of getting more heads would be 4/9 + 4/9 = 8/9 which is much higher than for a fair coin.

Problem 7 - From your answers to Problem 2, what would be the probability of getting only Us in 4 coin tosses if A) P(U) = 1/2? B) P(U) = 1/3?

Problem 8 - The fraction of the lunar surface that is cratered is 3/4, while the mare (dark areas) have few craters and occupy 1/4 of the surface area. If four asteroids were to strike the moon in 100,000 years, what is the probability that all four would strike the cratered areas?

Answer Key

Problem 1 - The 16 possibilities are as follows:

C U U U	C C U U	U C U C	C U C C
U C U U	C U C U	U U C C	U C C C
U U C U	C U U C	C C C U	C C C C
U U U C	U C C U	C C U C	U U U U

Note if there are two outcomes for each coin flip, there are 2 x 2 x 2 x 2 = 16 independent possibilities.

Problem 2 - There is only one outcome that has 'U U U U'

Problem 3 - There is only one outcome that has 'C C C C'

Problem 4 - From the tabulation, there are 6 ways to get this outcome in any order.

Problem 5 - There are 4 outcomes that have only one U out of the 16 possible outcomes, so the fraction is 4/16 or 1/4.

Problem 6 - The fraction is 6 / 16 reduces to 3/8 so the odds are 3 chances in 8.

Problem 7: A) There is only one outcome that has all Us, and if each U has a probability of 1/2, then the probability is 1 x (1/2) x (1/2) x (1/2) x (1/2) = 1/16.
B) If each U has a probability of 1/3, then the probability is (1/3) x (1/3) x (1/3) x (1/3) = 1/81.

Problem 8 - P(U) = 1/4 while P(C)= 3/4, so the probability that all of the new impacts are also in the cratered regions is the outcome C C C C which is (3/4) x (3/4) x (3/4) x (3/4) = 81 / 256.

Lunar Meteorite Impact Risks

A December 4, 2006 CNN.Com news story, based on the research by Bill Cooke, head of NASA's Meteoroid Environment Office suggests that one of the largest dangers to lunar explorers will be meteorite impacts. Between November 2005 and November 2006, Dr. Cooke's observations of lunar flashes (see image) found 12 of these events in a single year. The flashes were caused primarily by Leonid Meteors about 3-inches across, impacting with the equivalent energy of 150-300 pounds of TNT.

The diameter of the moon is 3,476 kilometers.

Problem 1: From the formula for the surface of a sphere, what is the area, in square kilometers, of the side of the moon facing Earth?

Problem 2: Although an actual impact only affects the few square meters within its immediate vicinity, we can define an impact zone area as the total area of the surface being struck, by the number of objects striking it. What was the average impact zone area for a single event?

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts?

Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Problem 6: The lunar image shows that the impacts are not really random, but seem clustered into three groups. Each group covers an area about 700 kilometers on a side. What is the average impact zone area for four strikes per zone?

Answer Key:

Problem 1: From the formula for the surface of a sphere, A) what is the area, in square kilometers, of the side of the moon facing Earth?

Answer: $2 \times 3.141 \times (1738\text{ km})^2 = 1.89 \times 10^7\text{ km}^2$

Problem 2: Answer: $1.89 \times 10^7\text{ km}^2 /12 = 1.58 \times 10^6\text{ km}^2$

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Answer: A) $S = (1.58 \times 10^6\text{ km}^2)^{1/2}$ about 1,257 kilometers B) 1,257 kilometers.

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts?

Answer: 1 year / 12 impacts = One month.

Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Answer: 1 impact per 1.58×10^6 square kilometers per month. The area of the horizon region around the colony is about $\pi \times (3\text{km})^2 =$ 27 square kilometers. This area is $1.58 \times 10^6\text{ km}^2 /27\text{ km}^2$ or about 60,000 times smaller that the average, monthly impact area. That suggests you will have to wait about 60,000 times longer than the time it takes for one impact or 60,000 months, which equals 5,000 years, assuming that the distribution of impacts is completely random, unbiased and has a uniform geographic distribution across the Moon's surface.

Problem 6: Answer: (700 km) x (700 km) / 4 = 1 impact per 122,500 km^2 zone area.
Horizon area = 27 km^2, so the impact zone area is 122,500 / 27 = 4,500 times larger. You would need to wait about 4,500 x 1 month or 375 years for an impact to happen within your horizon.

Note to Teacher: This calculation assumes that the clustering of impacts is a real effect that persists over a long time. In fact, this is very unlikely, and it is more statistically probable that when thousands of impacts are plotted, a more uniform strike distribution will result. This is similar to the result of flipping a coin 12 times and getting a different outcome than half-Heads and half-Tails.

The Lunar Impact Crater Size Distribution

This is an image of a region near the Moon's South Pole taken by the Clementine spacecraft. The image size is 375 km x 375 km.

Problem 1 – What is the scale of this image in kilometers/millimeter? What is the area, A, of this region in kilometers^2 ?

Problem 2 – Count the number of craters in this region within each of the size intervals listed in the table below. Calculate the crater area assuming craters are circles with an average diameter within each range.

Diameter Range (mm)	Diameter Range (km)	Average radius (km)	Number of Craters	Total Crater Area (km^2)
20 - 30	50 - 75	31		
10 - 19	25 - 49	18		
5 - 9	12 - 24	9		
3 - 4	8 - 11	5		

Problem 3 – What percentage of the photograph is covered by an impact crater?

Problem 1 – The scale of the image is 375 km / 150 mm = 2.5 km/mm. The area of the image is A = 375 km x 375 km **= 110,625 kilometers^2**

Problem 2 – Count the number of craters in this region within each of the size intervals listed in the table below. The figure below shows some example craters used in the table. Student numbers for the smallest craters may differ, but this could lead to a discussion about how to average all of the student data together to get a better estimate for each row in the table.

Diameter Range (mm)	Diameter Range (km)	Average radius (km)	Number of Craters	Total Crater Area (km^2)
20 - 30	50 - 75	31	4	4 x 3.14 x (31)2 = 12,070
10 - 19	25 - 49	18	8	8,136
5 - 9	12 - 24	9	20	5,080
3 - 4	8 - 11	5	18	283
				Total area = 25,269

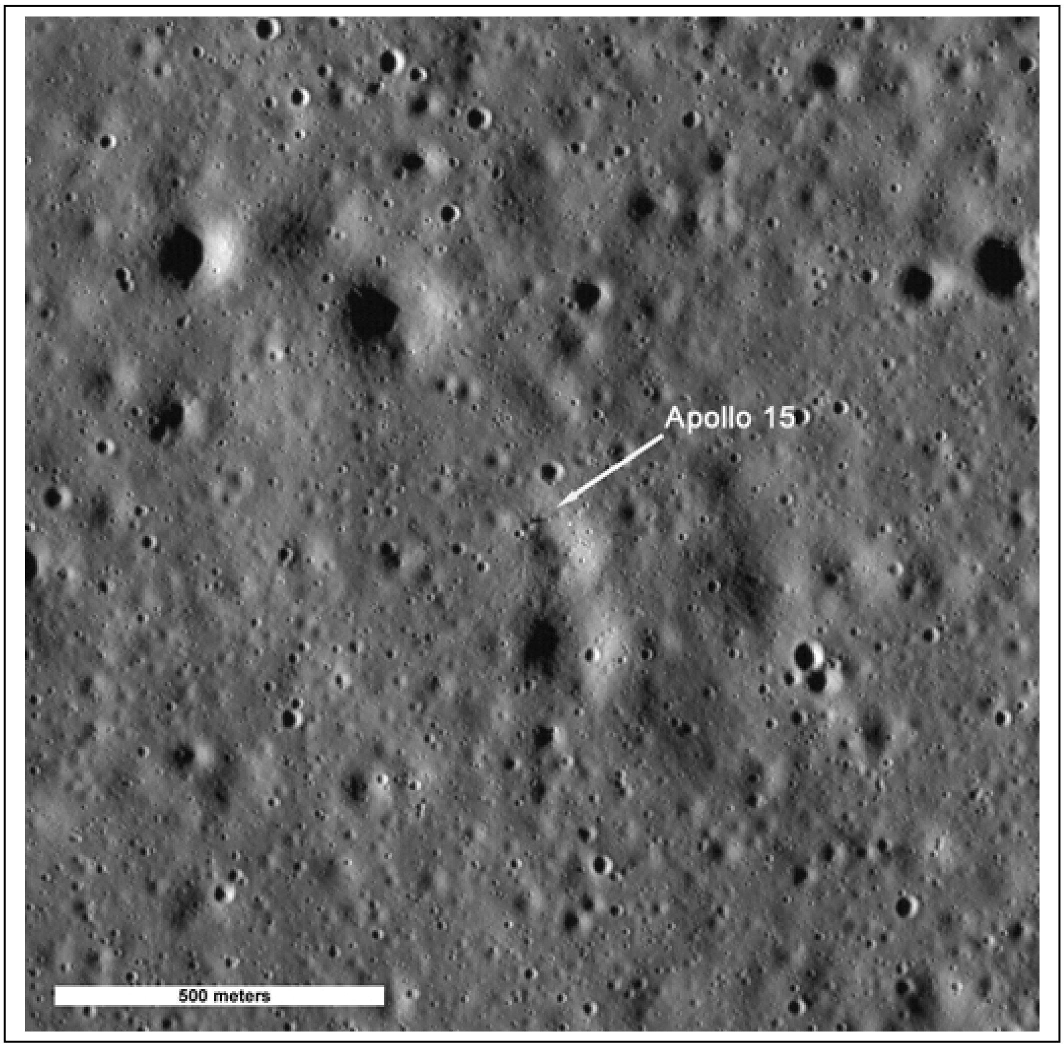
Problem 3 – The total cratered area from the table is 25,269 km^2, but the image has an area of 110,625 km^2 so the fraction covered by craters is 25269/110625 = 0.23 or **23%.**

The Relative Ages of Lunar Surfaces

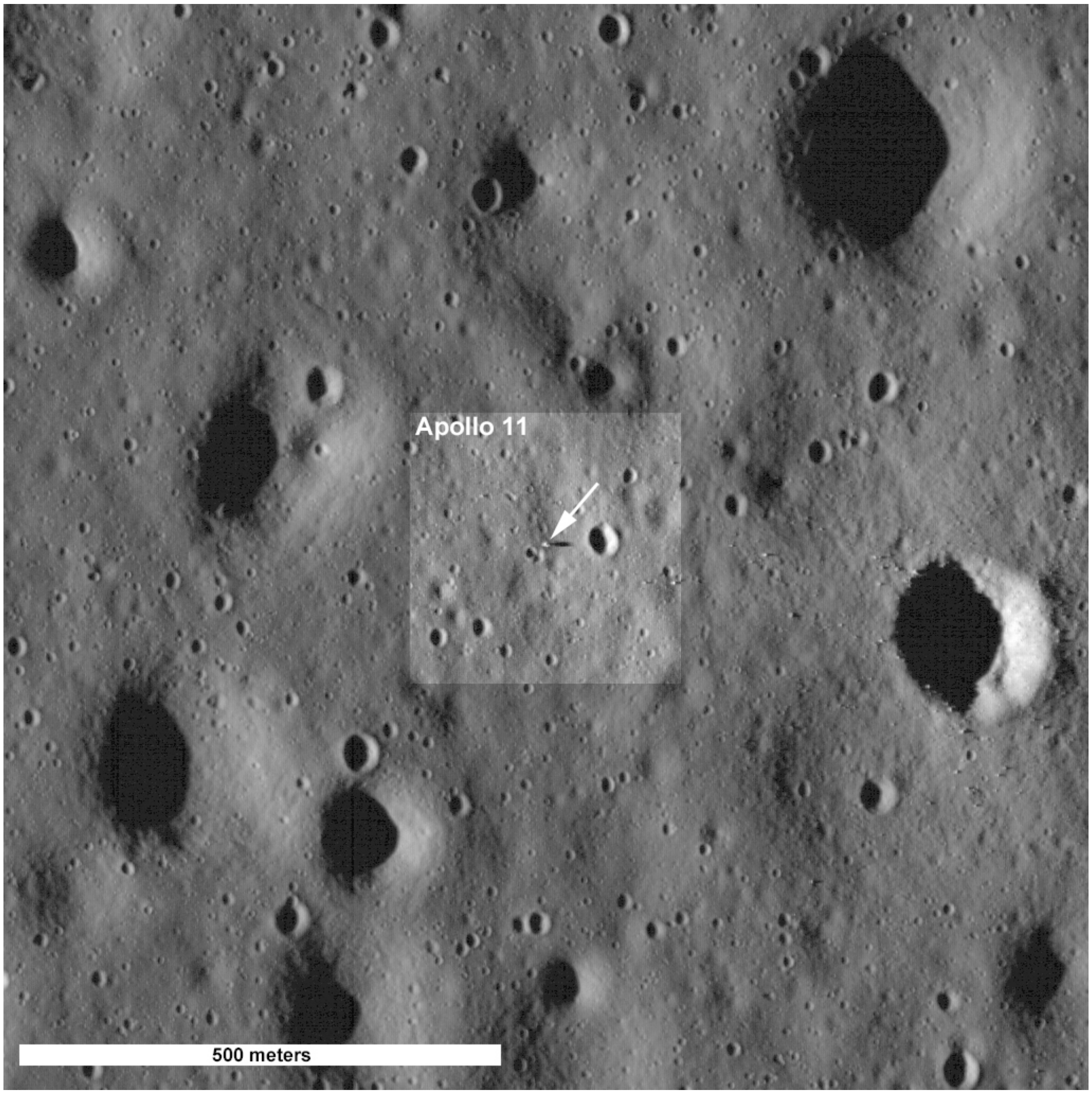
We have all sees pictures of craters on the moon. The images on the next two pages show close-up views of the cratered lunar surface near the Apollo 15 and Apollo 11 landing areas. They were taken by NASA's Lunar Reconnaissance Orbiter (LRO) from an orbit of only 25 kilometers!

Meteors do not arrive on the moon at the same rates. Very large meteors that produce the largest craters are much less common than the smaller bodies producing the smallest craters. That's because there are far more small bodies in space than large ones. Astronomers can use this fact to estimate the ages of various surfaces in the solar system by just comparing the number of large craters and small craters that they find in a given area.

Let's have a look the images below, and figure out whether Apollo-11 landed in a relatively younger or older region than Apollo 15!



This is the Apollo-15 landing area near the foot of the Apennine Mountain range. Note the bar indicating the 'scale' of the image. The arrow points to the location of the Lunar Descent Module.



This image taken by LRO is of the Apollo-11 landing area in Mare Tranquilitatus, with the arrow pointing to the Lunar Descent Module. The LDM was the launching platform for the Apollo-11 Lunar Excursion Module, which carried astronauts Neil Armstrong and Buzz Aldrin back to the orbiting Command Module for the trip back to Earth.

Note the length of the '500 meter' bar, which gives an indication of the physical scale of the image. How long would it take you to walk 500 meters?

Astronomers assume that during the last 3 billion years following the so-called 'Late Heavy Bombardment Era' the average time between impacts that created craters has been constant. That means that the more time that passes, the more craters you will find, and that they are produced at a more or less steady number for each million years that passes.

Dating a cratered surface.

For each of the above images, perform these steps.

Step 1 - With the help of a millimeter ruler, and the '500 meter' line in the image, calculate the scale factor for the image in terms of meters per millimeter.

Step 2 - Calculate the total area of the image in square kilometers.

Step 3 - Identify and count all craters that are bigger than 20 meters in diameter.

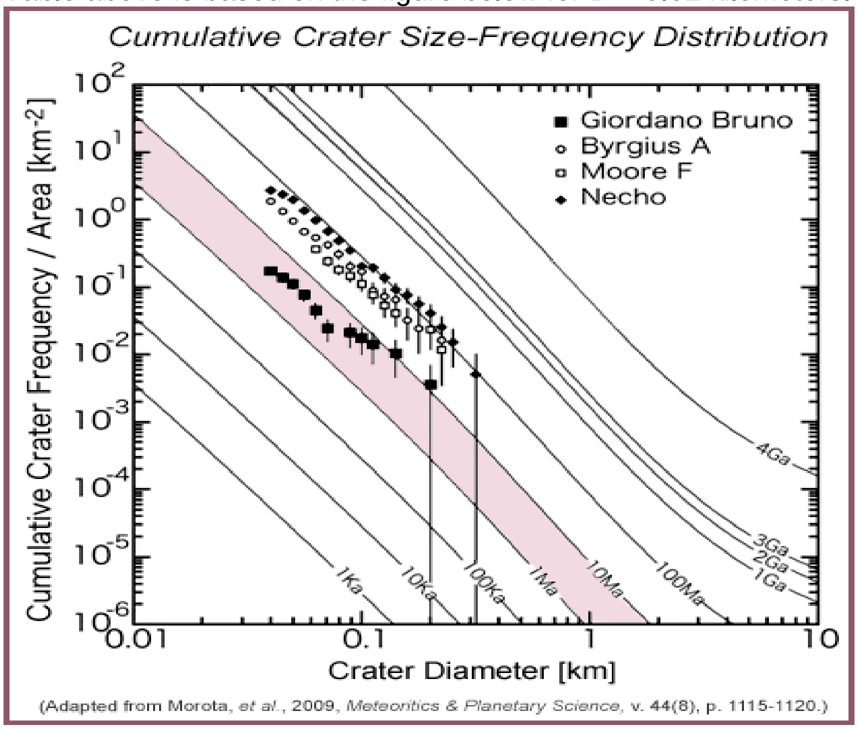
Step 4 - Divide your answer in Step 3 by the area in square kilometers in Step 2.

Step 5 - Look at the table below and estimate the average age of the surface.

Problem 1 - From your answer for each Apollo landing area in Step 5, which region of the moon is probably the youngest?

Estimated Age	Total number of craters per square kilometer
1000 years	0.0008
10000 years	0.008
100,000 years	0.08
1 million years	0.8
10 million years	8.0
100 million years	80.0
1 billion years	800.0

Table above is based on the figure below for D = 0.02 kilometers.



Another thing you might consider doing is to measure the diameters of as many craters as you can, and then plot a histogram (bar graph) of the number of craters you counted in a range of size intervals such as 5 - 10 meters, 11 to 20, 21 to 30 and so on. Because erosion (even on the moon!) tends to eliminate the smallest craters first, you can compare two regions on the moon in terms of how much erosion has occurred.

Problem 2 - Select the same-sized area on each of the Apollo images and count all the craters you can find within the size intervals you selected. How do the two landing areas compare to one another in terms of their crater frequency histograms?

Problem 3 - Which surface do you think has experienced the most re-surfacing or erosion?

Problem 4 - Without an atmosphere, winds or running water, what do you think could have caused changes in the lunar surface after the craters were formed?

Note to Teachers: More technical information on crater dating can be found at

"How young is the Crater Giordano Bruno"

http://www.psrd.hawaii.edu/Feb10/GiordanoBrunoCrater.html

Exploring the Moon's Orbit

Date	Distance (km)
1-17	369,882
2-11	367,919
3-10	362,399
4-7	358,313
5-6	356,953
6-3	358,482
7-1	362,361
7-29	367,317
8-23	369,730
9-19	365,748
10-17	360,672
11-14	357,360
12-12	357,073

Because the orbit of the Moon is an ellipse with Earth at one focus, the Moon is closer to Earth during a portion of each month, called perigee, and farthest from Earth some 2 weeks later called the apogee.

The phases of the Moon can occur at different points along this orbit, but are not fixed to specific spots along the orbit. Some full moons occur when the moon is at its apogee and others when it is at perigee. Because of this, some full moons will appear slightly larger when the moon is at perigee than when it is further away at apogee.

The table above gives the perigee distances of the moon during 2012.

Problem 1 – To the nearest kilometer, what is the average perigee distance?

Problem 2 – Subtract the average perigee distance from the individual perigee distance each month. What is the range of distances spanned by the perigees compared to their average value to the nearest kilometer?

Problem 3 – Full moons occurred on the following dates: 1-9, 2-7, 3-8, 4-6, 5-6, 6-4, 7-3, 8-2, 8-31, 9-30, 10-29, 11-28 and 12-28. Which of these full moons occurred at lunar perigee? These are called Perigean Full Moons.

Problem 1 – To the nearest kilometer, what is the average perigee distance?

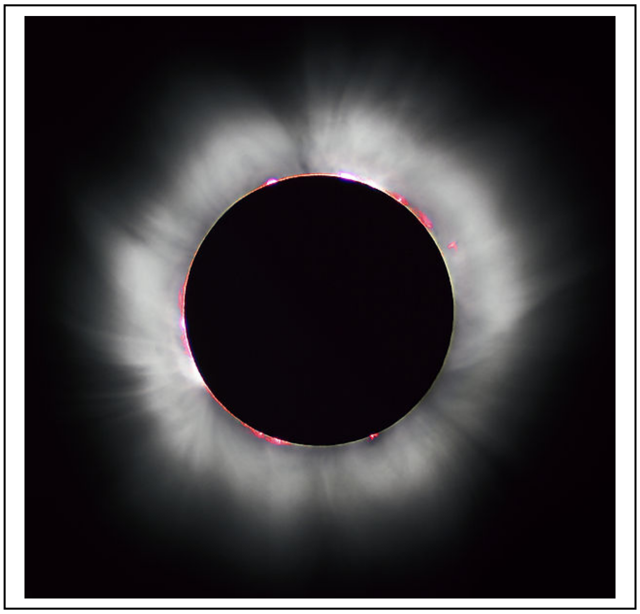
Answer: 4714209/13 = **362,631 km**.

Problem 2 – Subtract the average perigee distance from the individual perigee distance each month. What is the range of distances spanned by the perigees compared to their average value to the nearest kilometer?

Answer: The farthest perigee is for January 17 with 369,882 and the closest is for May 6 with 356,953 so subtracting the average of 362,631 km we get a range of -5,678 km to +7,251 km.

Problem 3 – Full moons occurred on the following dates: 1-9, 2-7, 3-8, 4-6, 5-6, 6-4, 7-3, 8-2, 8-31, 9-30, 10-29, 11-28 and 12-28. Which of these full moons occurred at lunar perigee? These are called Perigean Full Moons.

Answer: Only the May 6, 2012 full moon occurs on the perigee date. The others occur several days later or earlier in time.



Date	Minimum Perigee (km)
1-30-2010	356,592
3-19-2011	356,577
5-6-2012	356,953
6-23-2013	356,989
8-10-2014	356,896
9-28-2015	356,876
11-14-2016	356,511
5-26-2017	357,209
1-1-2018	356,565
2-19-2019	356,761
4-7-2020	356,908

The photo above is of a total solar eclipse photographed in France by Luc Viatour (www.lucnix.be). The exact angular diameter of the moon has to precisely match the same angular diameter of the sun in order for both disks to be the same size and for a total solar eclipse to occur. Typically, the angular size of the solar disc as viewed from Earth varies from 31.6 minutes of arc (0.52667 degrees) when Earth is farthest from the sun in June, to 32.7 minutes of arc (0.545 degrees) when Earth is closest in December.

Although the moon orbits the Earth every 27.3 days, and a lunar month equals 29.5 days, the minimum perigee distance during each orbit changes as the lunar orbit rotates in space. The table gives the smallest perigean distances for the 12-13 lunar perigee distances during each year.

The angular diameter of the moon can be found from its known diameter in kilometers (3475 km) and its distance to Earth from the formula:

Angle in minutes of arc = 3437.8 x Diameter/Distance.

Problem 1 – To the nearest hundredth of an arc minute, what is the angular diameter of the moon for the various perigean distances given in the table above?

Problem 2 – What are the mean, median and mode of the perigean angular sizes for the moon?

Angle in minutes of arc = 3437.8 x 3475 km/Distance.

Date	Minimum Perigee (km)	Moon Angular Diameter (minutes)
1-30-2010	356,592	33.50
3-19-2011	356,577	33.50
5-6-2012	356,953	33.47
6-23-2013	356,989	33.46
8-10-2014	356,896	33.47
9-28-2015	356,876	33.47
11-14-2016	356,511	33.51
5-26-2017	357,209	33.44
1-1-2018	356,565	33.50
2-19-2019	356,761	33.49
4-7-2020	356,908	33.47

Problem 1 – To the nearest hundredth of an arc minute, what is the angular diameter of the moon for the various Perigean distances given in the table above?

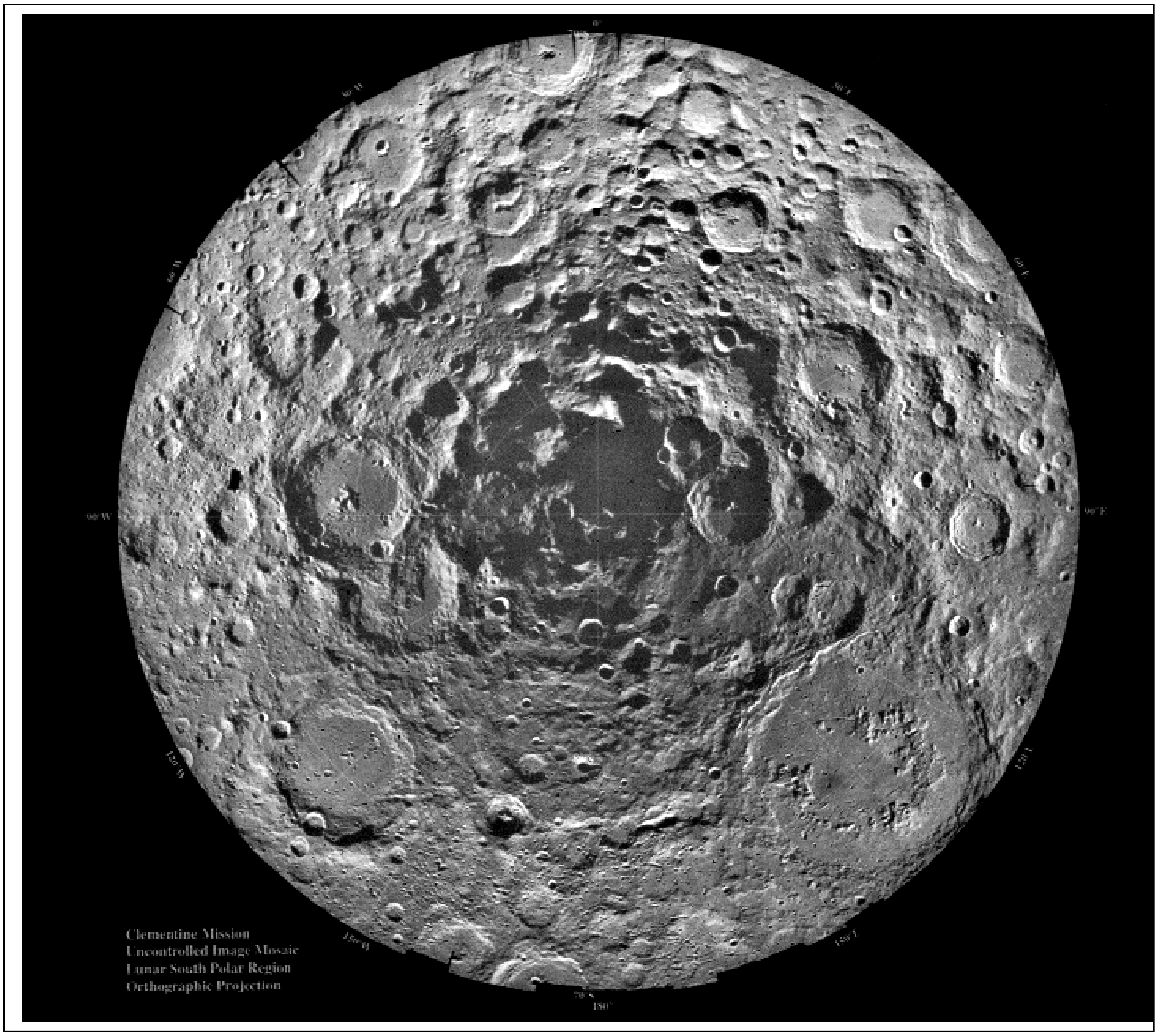
Answer shown in table.

Problem 2 – What are the mean, median and mode of the perigean angular sizes for the moon?

Answer: First sort the calculated values from smallest to largest.

1 33.44
2 33.46
3 33.47
4 33.47
5 33.47
6 33.47
7 33.49
8 33.50
9 33.50
10 33.50
11 33.51

Mean = 368.28/11 = **33.48**. **Median = 33.47** **Mode = 33.47**

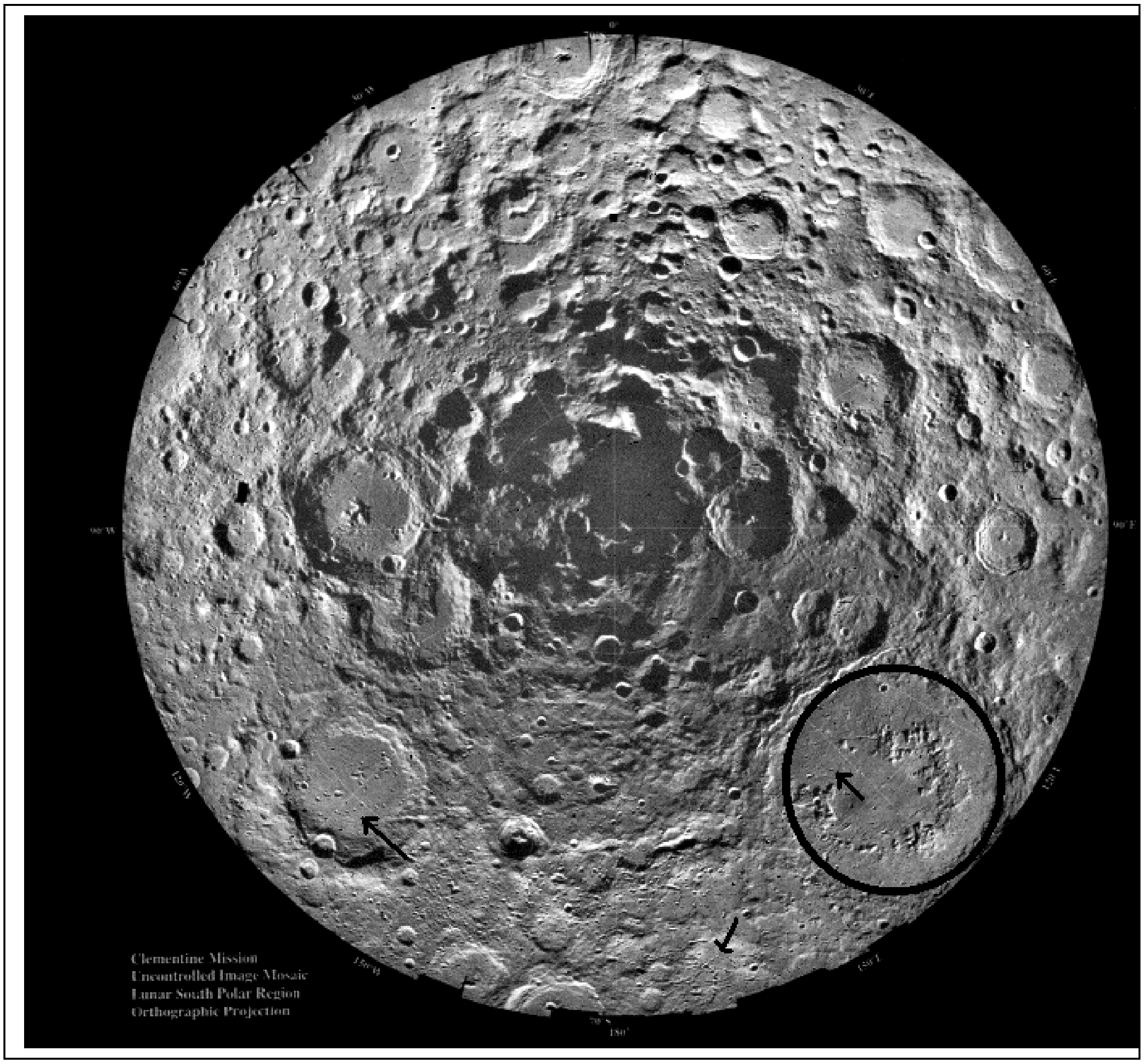


This is an image of the South Pole of the Moon taken by the Clementine satellite in 1996. The field has a diameter of 1,250 km. NASA's Lunar Reconnaissance Orbiter (LRO) will be launched in 2009. It will study the South Pole carefully, and at much higher resolution. Scientists believe the dark regions have been shadowed from sunlight for over 2 billion years. They are very cold places where we might expect to discover water ice. Water is very expensive to bring all the way from Earth. Future astronauts may land near these shadowed regions, set up a research station, and mine the ice to make drinking water and rocket fuel.

Problem 1 – Use a millimeter ruler to determine the scale of this image in kilometers per millimeter.

Problem 2 – What are the diameters of the largest and smallest craters, in kilometers, that you can find in this image?

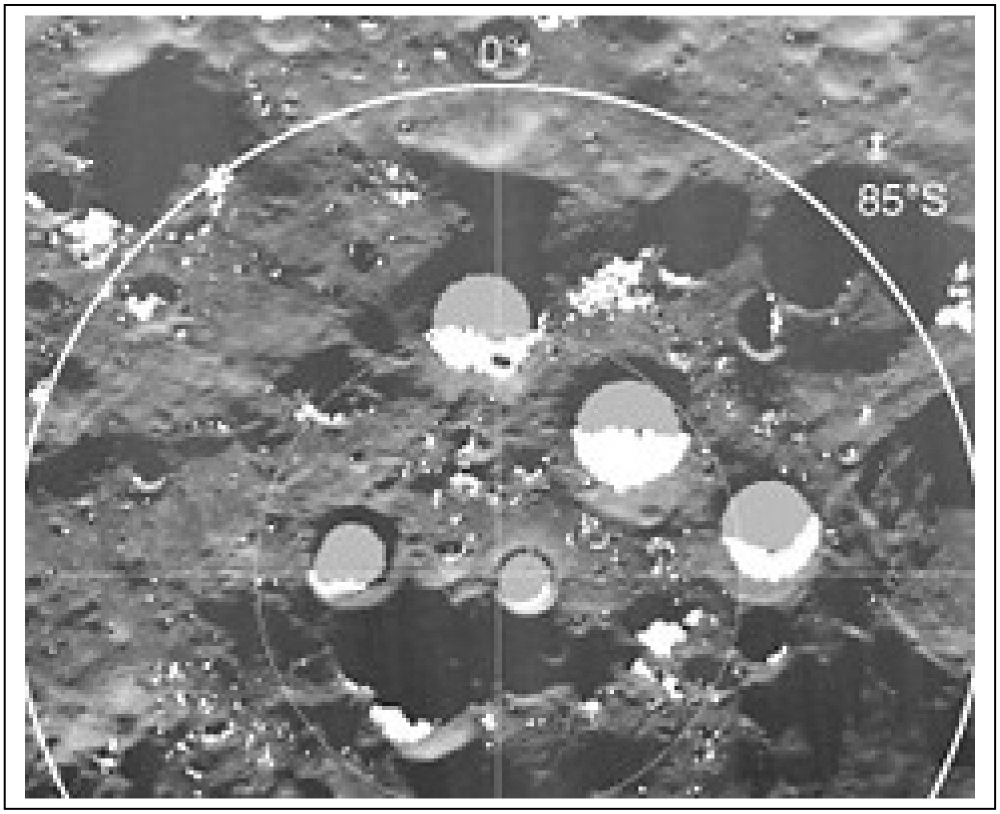
Problem 1 - If the first page is printed on a standard laser printer so that the page measures 8.5 x 11 inches (21.6 x 28.0 cm), the diameter of the lunar South Pole image is 143 mm. The information in the paragraph says that the diameter of the lunar image is 1,250 km, so the scale of this image is 1,250 km/ 143 mm = **8.7 km/millimeters.**



Problem 2 - Students should look at the image very carefully and find that the largest complete, crater is the one marked with the circle in the above image. It measures 30 mm across, and from the image scale of 8.7 km/mm, this corresponds to **261 km**!

There are many possibilities for the 'smallest' crater in this image. You have to look very closely at the picture to find small round marks. A few possibilities are shown with arrows in the picture above. They measure about 0.2 mm in diameter or **1.7 km.**

Estimating the Lunar South Pole Ice Resources

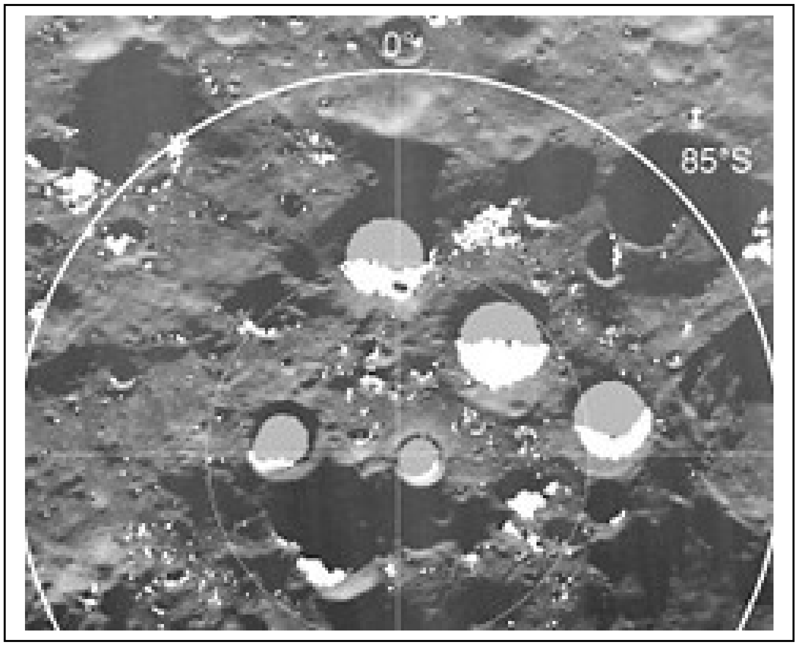


This is an image of the South Pole of the Moon revealed by Cornell University and Jet Propulsion Laboratory astronomers. They created this image by using the radar antennas of NASA's Deep Space Network at Goldstone, California. Instead of using visible light to penetrate the shadowed lunar craters, scientists used radio waves with a wavelength of 3.5 centimeters (8.6 gigahertz). Areas visible to the radar but in the sun's permanent shadow are marked in white. The floors of the five large craters are hidden from the sun's direct light and heat, and may be the largest potential deposits of water ice at the South Pole. The crater at the center of the two coordinate circles is Shackleton Crater and is almost directly at the Moon's South Pole, which is located where the vertical and horizontal lines cross.

<u>Problem 1</u> – If the portion of the large circle shown in the image has a radius of 160 kilometers, use a millimeter ruler to determine the scale of this image in kilometers per millimeter.

<u>Problem 2</u> – The radar map shows where conditions may be suitable for water ice to exist, without having evaporated into space due to solar heating. If the ice is mixed with lunar soil in equal parts, to a depth of 100 meters in each shadowed crater basin, what is the total volume of water ice, in cubic kilometers, that could exist in these craters?

Problem 1 - If the first page is printed on a standard laser printer so that the page measures 8.5 x 11 inches (21.6 x 28 cm), the radius of the lunar South Pole image is 65 mm. The information in the paragraph says that the radius of the lunar image is 160 km, so the scale of this image is 160 km/ 65 mm = **2.5 km/millimeters.**



Problem 2 - Students will calculate the combined surface areas, in square kilometers, of the circular white regions in each of the five craters, then multiply by the depth of the ice given in the problem as 100 meters or 0.1 kilometers. Remember to use the radius of the crater to calculate its area.

Crater 1 diameter = 7 mm = 17.5 km. Area = π x $(8.75)^2$ = 240.4 kilometers^2
Crater 2 diameter = 5 mm = 12.5 km. Area = π x $(6.25)^2$ = 122.7 kilometers^2
Crater 3 diameter =10 mm = 25 km. Area = π x $(12.5)^2$ = 490.7 kilometers^2
Crater 4 diameter =11 mm = 27.5 km. Area = π x $(13.75)^2$ = 593.7 kilometers^2
Crater 5 diameter =10 mm = 25 km. Area = π x $(12.5)^2$ = 490.7 kilometers^2

The total shadowed area is then **1932.2 kilometers^2**

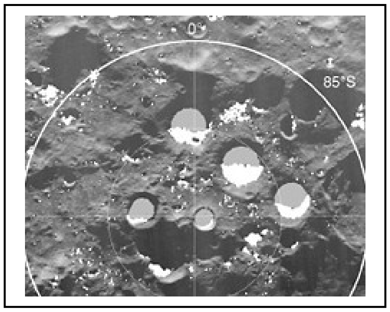
The volume (assuming a cylindrical volume) is

Area x Depth = 1932.2 kilometers^2 x 0.1 kilometers = **193.2 kilometers^3**

Because the ice is mixed with equal parts of lunar soil, only half of this total volume is actually water ice, so

Maximum water-ice volume = 193.2 x 0.5 = **96.6 kilometers^3**

The Shackleton Crater



This is an image of the Shackleton Crater at the Moon's South Pole. The photo at the upper left shows a radio wave image of this cratered and heavily-shadowed region and the location of Shackleton compared to other craters in the region. This image was taken by the Advanced Moon Imaging Experiment (AMIE) on board the European, Small Missions for Advanced Research in Technology (SMART-1) spacecraft on January 13, 2006. The Shackleton Crater photo was taken from a distance of 646 kilometers above the lunar surface.

Problem 1 – If the Shackleton Crater is 19 kilometers in diameter, use a millimeter ruler to determine the scale of this image in meters per millimeter. What is the physical size of the smallest detail you can see in the image?

Problem 2 - From your previous answer, what kinds of familiar objects have about the same size as the smallest features in this image?

Problem 3 - If this image had a size of 512 x 512 pixels, what is the resolution of this image in meters per pixel?

Problem 4 - A spacecraft has a size of 5 meters, and requires a smooth surface to land, with no boulders larger than 1 meter. What must be the resolution of the survey image to find a safe landing spot?

Problem 1 – A millimeter ruler placed across the shadowed crater measures 75 mm. The photo scale is therefore, 19,000 meters / 75 mm = **253 meters / millimeter.**

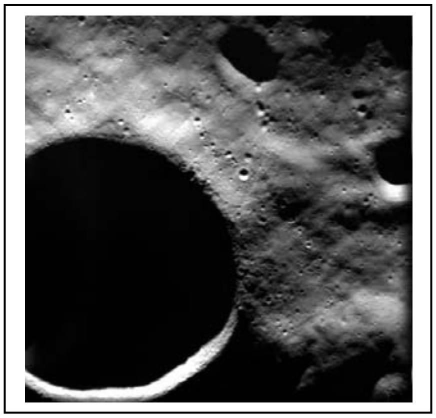
Problem 2 - The smallest feature in the photo is about 0.3 millimeters across. From the image scale of 253 meters/mm the feature is about **0.3 x 253 = 76 meters across**. This is about as large as a house or the lanes on a highway.

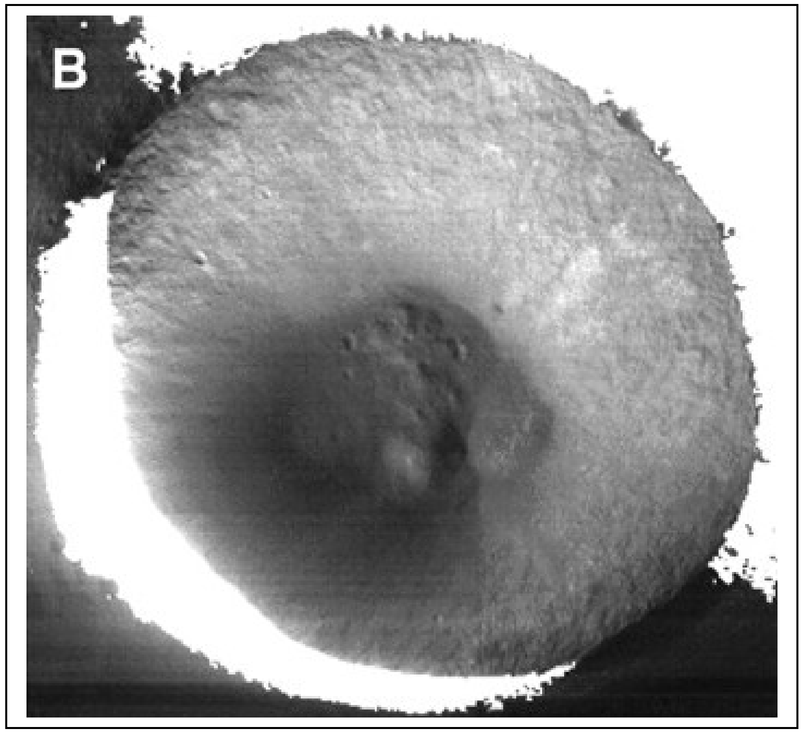
Problem 3 - The picture is 108 millimeters wide, which equals 512 pixels so each pixel is about 108/512 = 0.2 millimeters wide. From the image scale, this equals 0.2 x 253 = **50 meters per pixel.**

Problem 4 - If the surface has a lot of boulders larger than 1 meter, the spacecraft could tip over if it landed on one of these surface regions. We need to be able to see details in our images that are 1 meter in size so that we can avoid any large rocks or boulders at the landing site. This means we need a picture with a resolution of 1 meter per pixel, which is 50 times higher than the photo in this problem!

The Lunar Reconnissance Orbiter, to be launched in 2009, will photograph selected areas of the lunar surface at a resolution of 0.5 meters per image pixel.

Looking Inside the Shackleton Crater





This is an image of the Shackleton Crater at the Moon's South Pole. The photo at the upper left was taken by European, Small Missions for Advanced Research in Technology (SMART-1) spacecraft on January 13, 2006. The larger image was taken by the Terrain Camera onboard the Japanese Selenological and Engineering Explorer (SELENE) spacecraft in 2008. The Terrain Camera photographed the shadowed crater basin, which is faintly illuminated by reflected sunlight from the bright, and over-exposed, crater wall seen at the lower-left. (*Science*, November 7, 2008 v. 322 pp 938)

<u>Problem 1</u> – If the Shackleton Crater is 19 kilometers in diameter, use a millimeter ruler to determine the scale of the SELENE image in meters per millimeter. What is the physical size of the smallest detail you can see in the image?

<u>Problem 2</u> – The image is 350 x 350 pixels. What is the resolution of this picture in meters/pixel?

<u>Problem 3</u> - The scientists who analyzed the SELENE data measured the reflectivity of the lunar soil inside the crater and concluded that the crater floor is too dim to be covered by bare water ice. What other possibilities could exist for finding water ice in this crater?

Answers and Tips

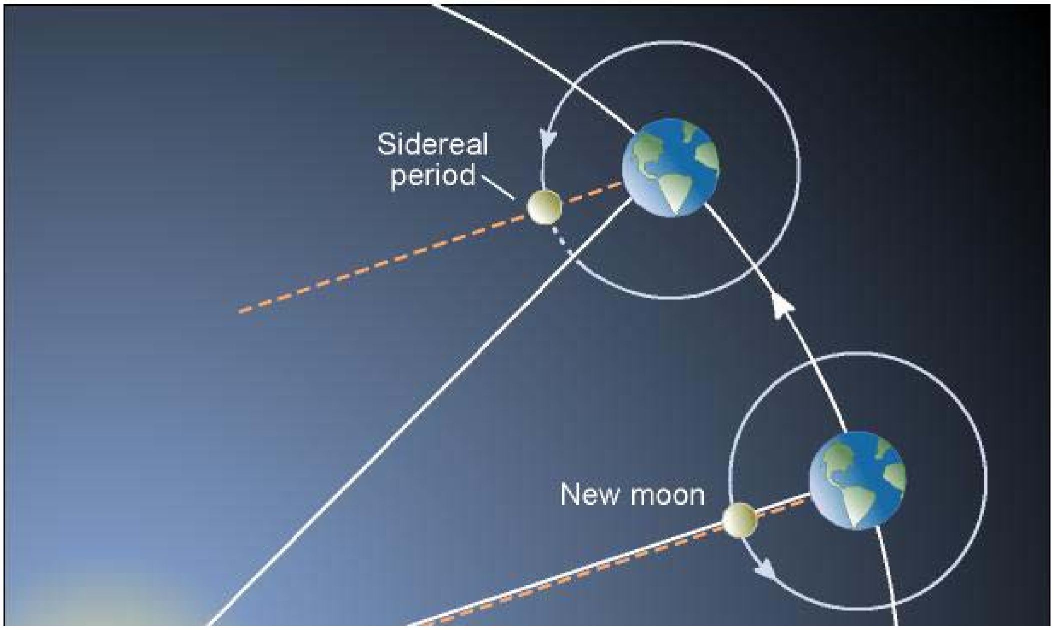
Problem 1 – The ruler would measure a diameter of about 90 millimeters. The image scale is then 19,000 meters / 90 mm = **211 meters/mm.**

What is the physical size of the smallest detail you can see in the image? If you look carefully, you will find features as small as 0.5 millimeters, which equals a physical size of 0.5 x 211 = **105 meters.**

Problem 2 – The width of the picture is 99 millimeters which equals 350 pixels, so 99 mm/350 pixels = 0.28 mm/pixel. From the image scale, this equals 0.28 mm x 211 m/mm = **59 meters/pixel.**

Problem 3 - Here are three possibilities:
1) The ice could be located under the lunar soil layer inside the crater.
2) The soil could be mostly made of rock with a small amount of water ice mixed into it.
3) The ice could be in the form of small rocks that are too small to see in the photograph but that cover the surface and are coated with lunar dust to hide their bright surfaces.

The Lunar Synodic and Sidereal Periods



The time between full moons, called the synodic period or synodic month, is 29.5 days, but this is not how long it takes the moon to go once around Earth in its orbit, which is called the sidereal period. The figure above, with the lower new moon drawn, shows that, new moons happen when the moon is along the line between the center of Earth and the center of the Sun. When the next new moon happens, Earth has moved along its orbit, and the moon has to travel a bit more than a full orbit to catch up to its next new moon position after 29.5 days. We can use this geometric information to figure out how long the moon actually takes to orbit earth once.

Problem 1 – Earth goes once around the sun every 365.24 days. By what angle does it travel along its orbit after 29.5 days?

Problem 2 – How many degrees along its orbit does the moon have to travel between New Moons?

Problem 3 – How long will it take the moon to travel the extra 29.1 degrees?

Problem 4 – How long does it take the moon to go once around in its orbit?

Problem 1 – Earth goes once around the sun every 365.24 days. By what angle does it travel along its orbit after 29.5 days?

Answer: 365 x 29.5/365.24 = **29.1 degrees**

Problem 2 – How many degrees along its orbit does the moon have to travel between New Moons?

Answer: Students should be able to see from the diagram that as Earth moves along its orbit by a specific number of degrees, this is the same angle as measured from the center of Earth, as the one between the next new moon location it its orbit, and the point where the moon completes a full orbit.

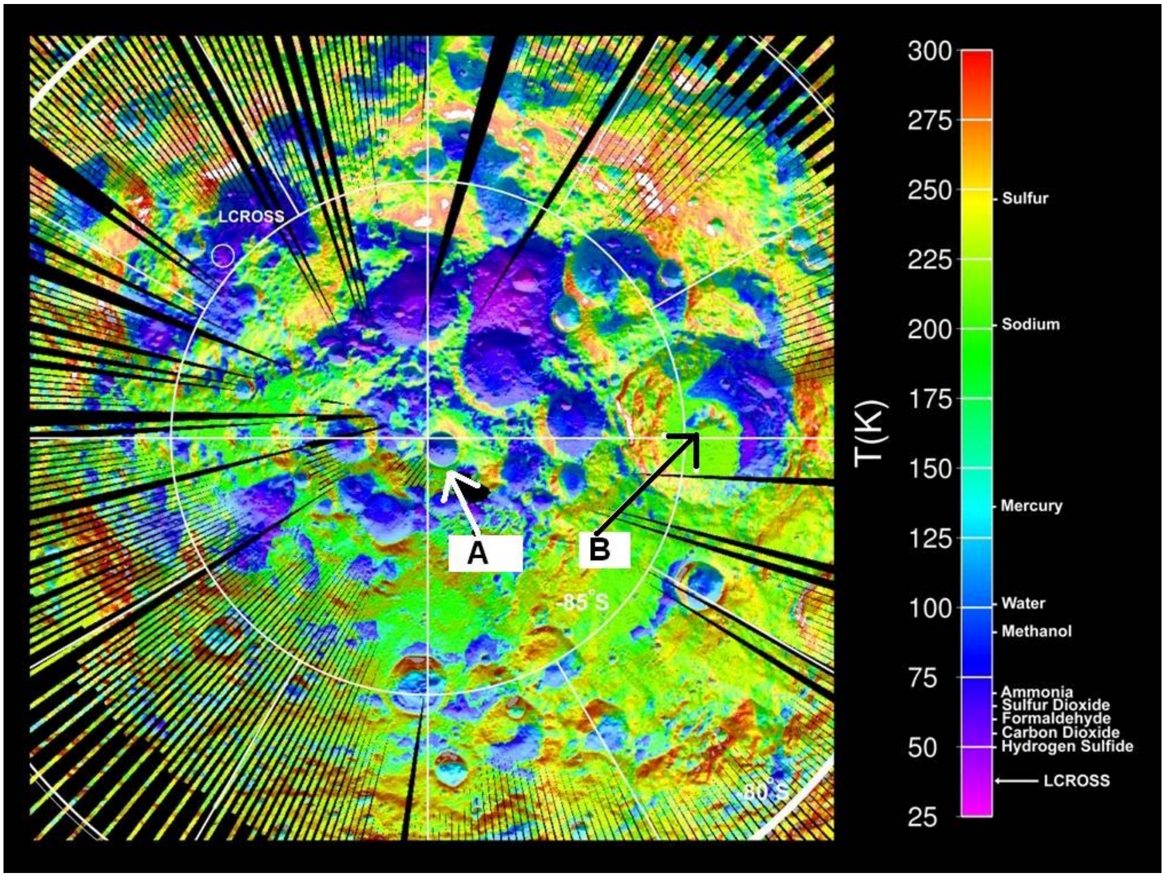
According to the figure, 360 degrees + 29.1 degrees = **389.1 degrees**.

Problem 3 – How long will it take the moon to travel the extra 29.1 degrees?

Answer: 389.1 degrees/29.5 days = 13.2 degrees per day, so it will take 29.1 degrees/13.2 = **2.2 days**.

Problem 4 – How long does it take the moon to go once around in its orbit?

Answer: 29.5 days – 2.2 days = **27.3 days**



The Lunar Reconnaissance Orbiter (LRO) has recently created the first surface temperature map of the south polar region of the moon using date taken between September and October, 2009 when south polar temperatures were close to their annual maximum values. The colorized map shows the locations of several intensely cold impact craters that are potential cold traps for water ice as well as a range of other icy compounds commonly observed in comets. The approximate maximum temperatures at which these compounds would be frozen in place for more than a billion years is shown on the scale to the right. The LCROSS spacecraft was targeted to impact one of the coldest of these craters, and many of these compounds were observed in the ejecta plume. *(Courtesy: UCLA/NASA/JPL)*

Problem 1 - The width of this map is 500 km. What are the diameters of Crater A (Shackleton) and Crater B (Amundsen) in kilometers?

Problem 2 - In which colored areas might an astronaut expect to find conditions cold enough to recover all of the elements and molecules indicated in the vertical temperature scale to the right?

Problem 3 - The Shackleton Crater (Crater A) is cold enough to trap water and methanol. From Problem 1, and assuming that the thickness of the water deposit is 100 meters, and occupies 10% of the volume of the circular crater, how many cubic meters of water-ice might be present?

Problem 1 - The width of this map is 500 km. What are the diameters of Crater A (Shackleton) and Crater B (Amundsen) in kilometers?

Answer: If the page is printed as 8.5 x 11-inches, the width of the colorized image is 106 mm wide, which corresponds to 500 km, so the image scale is 500 km/106mm = 4.7 km/mm. Crater A has a diameter of 4.4 mm so its actual diameter is 4.4 mm x (4.7 km/mm) = **20 kilometers**. Crater B has a diameter of 34 mm, so its actual diameter is about 34 x 4.7 = **160 kilometers**.

Problem 2 - In which colored areas might an astronaut expect to find conditions cold enough to recover all of the elements and molecules indicated in the vertical temperature scale to the right?

Answer: Students should note that all compounds <u>above</u> a given temperature will be present in conditions cold enough to 'trap' these compounds. **Areas where the temperature is below 50 K will be cold enough to trap all of the identified compounds. These are areas colored lavender at the bottom of the temperature scale, and in areas similar to where LCROSS impacted.**

Problem 3 - The Shackleton Crater (Crater A) is cold enough to trap water and methanol. From Problem 1, and assuming that the thickness of the water deposit is 100 meters, and occupies 10% of the volume of the circular crater, how many cubic meters of water-ice might be present?

Answer: The diameter of the circular crater is 20 km or 20,000 meters, so its radius is 10,000 meters. The area of the crater is then $A = \pi (10,000)^2 = 314,000,000$ meters^2. Since the thickness of the deposit is perhaps 100 meters, the volume of this disk of material is V = 100 meters x 314,000,000 meters^2 = 3.1×10^{10} meters^3. Since only 10% of this is hypothetically water-ice, the ice volume is just 0.1V or **3.1×10^9 meters^3.**

Note: This ice volume is similar to a small glacier!

Apollo 17 - Launching from the Moon's Surface

On December 14, 1972 at 10:54:37 p.m. GMT, Astronauts Eugene Cernan and Harrison Schmidt blasted off from the lunar surface in the Lunar Module (LM). The launch was recorded by a camera left behind at the landing site in the Taurus-Litrow region. A sequence of images from this recording is shown to the left.

The sequence of images runs from the top to the bottom. The top image was taken at 10:54:37.00 p.m. and the bottom image was taken at 4.9 seconds later at 10:54:41.87 p.m. The width of the LM is 4.3 meters. See the YouTube video of the LM launch at http://www.youtube.com/watch?v=iziumcklDbM&feature=related

Table of LM Heights and Times

Image	Time (seconds)	Height (meters)	Speed (m/s)
1	0	0	
2	1.8	2	
3	2.3	6	
4	2.9	10	
5	3.2	15	
6	3.7	18	
7	4.2	21	

Problem 1 - What is the average speed of the LM during the 4.87 seconds covered by this image sequence?

Problem 2 - What are the average speeds of the LM between Image 1 and A) Image 2? B) Image 3? C) Image 4? D) Image 5? E) Image 6? F) Image 7? Enter these speeds in the above table.

Problem 1 - What is the average speed of the LM during the 4.87 seconds covered by this image sequence?

Answer: The distance traveled was 21.0 meters, which took 4.9 seconds, so the average speed was S = 21.0 m/4.9s so **S = +4.3 meters/sec**.

Problem 2 - What are the average speeds of the LM between Image 1 and A) Image 2? B) Image 3? C) Image 4? D) Image 5? E) Image 6? F) Image 7? Enter these speeds in the table.

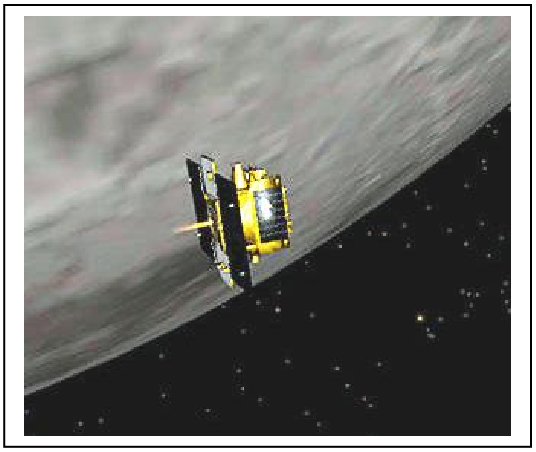
Answer: A) distance traveled = 2 meters, time = 1.8 seconds, so speed = 2/1.8 = **1.1 meters/sec**.
B) distance traveled = 6 meters, time = 2.3 seconds, so speed = 6/2.3 = **2.6 meters/sec.**
C) distance traveled = 10 meters, time = 2.9 seconds, so speed = 10 / 2.9 s = **3.4 meters/sec**.
D) distance traveled = 15.0 meters, time = 3.2 seconds, so speed = 15m / 3.2s = **4.7 meters/sec**.
E) distance traveled = 18.0 meters, time = 3.7 seconds, so speed = 18.0 m / 3.7 s = **4.9 meters/sec**.
F) distance traveled = 21.0 meters, time = 4.2 seconds, so speed = 21 m / 4.2 s = **5.0 meters/sec**.

Table of LM Heights and Times

Image	Time (seconds)	Height (meters)	Speed (m/s)
1	0	0	
2	1.8	2	1.1
3	2.3	6	2.6
4	2.9	10	3.4
5	3.2	15	4.7
6	3.7	18	4.9
7	4.2	21	5.0

Note: These speeds are approximate due to the quality of the video images, which had no time stamps to verify when the individual frames were taken. The times and heights were estimated from an approximate analysis of the video sequence.

The Grail and LRO Spacecraft Encounters in Lunar Orbit



On May 31, 2012 the Grail 'Ebb' spacecraft and the Lunar Reconnaissance Orbiter (LRO) will come very close to each other in their orbits around the moon. LRO is in a polar orbit, while Grail Ebb is in an equatorial orbit. Although there is no scientific value in the encounter, it does represent one of the first times that two NASA spacecraft orbiting the same astronomical body have passed so close to each other, and with the capability of actually seeing each other.

The table to the left gives the encounter times in the afternoon (Eastern Standard Time in hours, minutes and seconds) and distances (in kilometers) between the spacecraft.

Time	Distance	Time	Distance
4:00:38	190	4:01:23	110
4:00:42	180	4:01:35	105
4:00:46	170	4:01:46	110
4:00:51	160	4:01:56	120
4:00:56	150	4:02:03	130
4:01:01	140	4:02:09	140
4:01:07	130	4:02:14	150
4:01:14	120	4:02:20	160

Problem 1 – At what time were the spacecraft at their closest distances from one another?

Problem 2 – About how fast, in kilometers/hour was the distance between them changing just before closest approach?

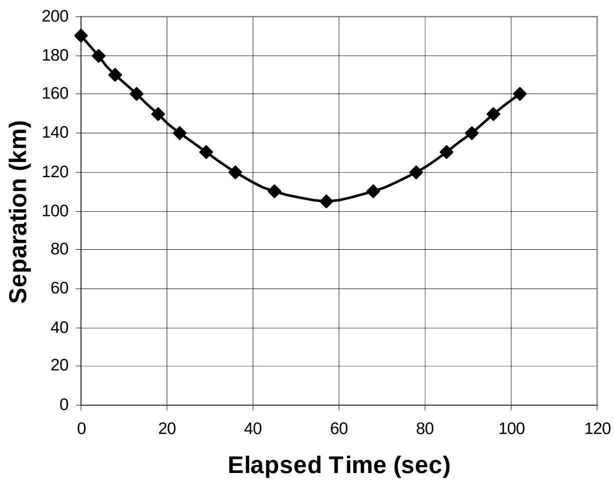
Problem 3 - Calculate the elapsed time of the encounter since 4:00:38 in seconds. Graph the tabular data in terms of elapsed time in seconds and distance in kilometers. What shape does the plotted curve resemble?

Problem 4 - The Grail 'Ebb' spacecraft will attempt to take a picture of LRO. At a distance of 50 kilometers, Grail/Ebb can just resolve an object if it is 8-meters across. That means that the angle corresponding to 8 meters at a distance of 50 kilometers is just large enough to be discerned by Grail. The LRO spacecraft is about 4 meters across. Using simple proportions, and the fact that the angular size of an object is inversely proportional to its distance, will Grail be able to see any details on the LRO spacecraft at the time of closest approach?

Problem 1 – At what time were the spacecraft at their closest distances from one another? Answer: This would be at **4:01:35** at a distance of 105 kilometers.

Problem 2 – About how fast, in kilometers/hour was the distance between them changing just before closest approach? Answer: Speed = distance/time. Distance = 110-105 = 5 km; time = 4:01:35 – 4:01:23 = 12 seconds so speed = 5km/12sec = 0.417 km/sec. In terms of km/hr, speed = 0.41 km/s x (3600 sec/1 hr) = **1500 km/hr**.

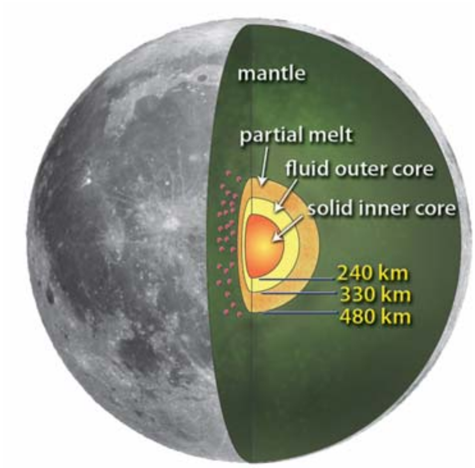
Problem 3 - Calculate the elapsed time of the encounter since 4:00:38 in seconds. Graph the tabular data in terms of elapsed time in seconds and distance in kilometers. What shape does the plotted curve resemble? Answer: **A parabola!**



Problem 4 - The Grail 'Ebb' spacecraft will attempt to take a picture of LRO. At a distance of 50 kilometers, Grail/Ebb can just resolve an object if it is 8-meters across. That means that the angle corresponding to 8 meters at a distance of 50 kilometers is just large enough to be discerned by Grail. The LRO spacecraft is about 4 meters across. Using simple proportions, and the fact that the angular size of an object is inversely proportional to its distance, will Grail be able to see any details on the LRO spacecraft at the time of closest approach?

Answer: At their closest separation, 105 kilometers is about twice as large as 50 km, and so for the same angle as seen by an 8 meter object at 50 km, the object would have to be about 8 x 105 km/50 km = 16.8 meters long in order to be seen by Grail. But LRO has a maximum size of only 4 meters, so that means Grail will only see LRO as an unresolved 'dot' of light as it passes by.

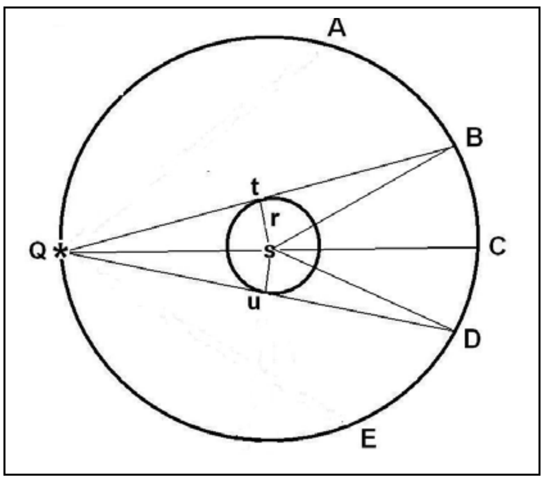
Note: Explore the encounter by using NASA's Eyes on the Solar System orbit simulator. Select the Moon, and the date and time of the encounter, then manipulate the scene until the two spacecraft orbits are highlighted. Step the time forward until you come to the encounter scene. To jump to this scene from here click on this link after first setting up EOSS to run on your computer: **http://1.usa.gov/LoD2YE**



Sometimes old data can uncover new secrets! Four seismometers were deployed by Apollo astronauts between 1969 and 1972. They were able to record continuous lunar seismic activity until late-1977.

A detailed mathematical analysis of this data reveals that the seismic data are consistent with a model in which the moon has a solid, iron-rich inner core and a fluid, primarily liquid-iron outer core. The core contains a small amount of elements such as sulfur, which is a composition similar to the core of our own Earth.

The analysis they used can be shown with a simple geometry problem:



Suppose a 'moonquake' occurs at Point Q in the figure. Its shock waves travel along the chords from Q to the various seismometers located at points A, B, C, D and E. In addition:

1 - The radius of the moon, R has a length of 1,738 km.

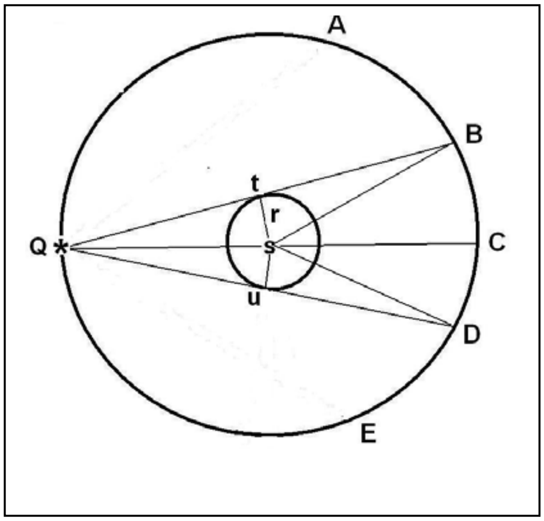
2 - The radius of the core, r, is defined by the segment **ts**.

3 - Angles **Qts** and **Qus** are right-angles.

Basic Seismology: An earthquake generates two kinds of shock wave signals called P-waves and S-waves. When rock is compressed like a sound wave, it produces a pressure wave called the P-wave along its direction of travel. When it moves from side-to-side perpendicular to its direction of travel, it is called a shear wave or S-wave. Although P-waves can travel through a liquid, S-waves are strongly reduced in strength, or sometimes absent all-together.

Suppose on the moon, Stations A and E record normal seismic S and P-wave signals, however, Stations B and D record signals in which the S-wave is slightly reduced in strength compared to Station's A and E. Station C records P-waves but no S-waves. Assume that Station C is in the shadow zone of the liquid lunar core, and that Stations B and D define seismic signals grazing the outer edge of a hypothetical lunar liquid core. Stations B and D are separated by 900 km along the lunar surface.

Problem 1 - From the figure above, and your knowledge of the properties of inscribed arcs what is the radius in kilometers of the core, r, based on this seismic date?



The arc, BCD = 900 km.
Qs = R = 1,738 km

Angle BsD = 360 x (900 km/ $2\pi R$)
= 360 x (900 km / 10918 km)
= 30 degrees

Angle BsC = 15 degrees

Angle BQC = 1/2 Angle BsC = 7.5 degrees

Angle BQC = Angle tQs

Segment st = r

Then:

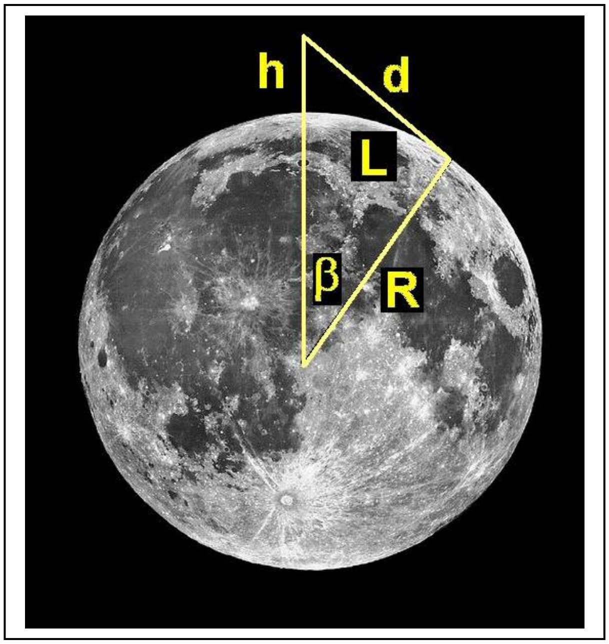
Sin (Angle tQs) = r/R

r = R sin(7.5 degrees)

r = 1,738 (0.13)

r = 226 km.

Calculating the Distance to the Horizon



An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna in order to insure proper reception out to a specified distance. This is especially important for lunar explorers because the Moon does not have an ionosphere capable of 'bouncing' the radio signals over the horizon.

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure above, and the Pythagorean Theorem, to derive the formula for the line-of-sight horizon distance, d, to the horizon tangent point.

Problem 2: Derive the distance along the planet, L, to the tangent point.

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6378 km); B) The Moon (1,738 km)

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance on the Moon?

Answer Key:

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure to the left to derive the formula for the line-of-sight horizon distance, D.

Answer: By the Pythagorean Theorem $d^2 = (R+h)^2 - R^2$

so $d = (R^2 + 2Rh + h^2 - R^2)^{1/2}$ and so the answer is $d = (h^2 + 2Rh)^{1/2}$

Problem 2: Derive the distance along the planet, l, to the tangent point.
Answer: From the diagram, $\text{Cos}(\beta) = R/(R+h)$ and so $L = R\,\text{Arccos}(R/(R+h))$

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) The Moon (1,738 km)

Answer: Use the equation from Problem 1.
A) R=6378 km and h=2 meters so
$d = ((2 \text{ meters})^2 + 2 \times 2 \text{ meters} \times 6.378 \times 10^6 \text{ meters}))^{1/2}$ = 5051 meters or 5.1 kilometers.

B) For the Moon, R=1,738 km so d = 2.6 kilometers

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance on the Moon?

Answer: A) h = 50 meters, R=1,738 km so d = 13,183 meters or 13.2 kilometers.

The moon is slowly pulling away from Earth. In the distant future, it will be much farther away from us than it is now. It is currently moving away at a rate of 3.8 centimeters per year. The following formula predicts the distance of the moon for a period extending up to 2 billion years from now:

$$D(T) = 38\,T + 385{,}000$$

where T is the elapsed time from today in millions of years, and D is the distance in kilometers

Problem 1 - Graph the function D(T) over the domain T:[0.0 , 2,000].

Problem 2 - What is the slope of the function?

Problem 3 - What is the Y-intercept for the function?

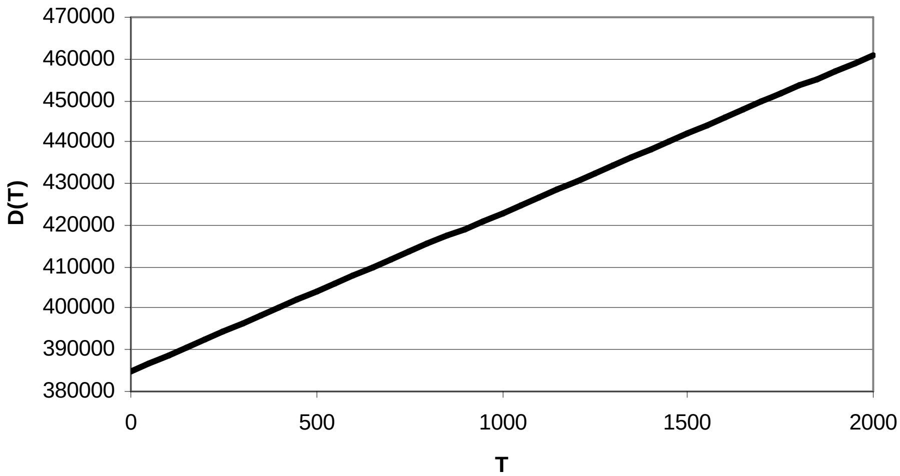
Problem 4 - What is the range of D(T) for the given domain?

Problem 5 - How many years in the future will the orbit be exactly D(T) = 423,000 kilometers?

Problem 6 - How far from Earth will the Moon be by 500 million years from now?

Problem 7 - How far will the moon be from Earth by T = 3,000?

Problem 1 - Graph the function D(T) over the domain T:[0.0 , 2,000].



Problem 2 - What is the slope of the function?
Answer: From the equation, which is of the form y = mx + b, the slope
M= 38 kilometers per million years.

Problem 3 - What is the Y-intercept for the function? Answer: For T = 0, the y-intercept, D(0) = **385,000 kilometers**.

Problem 4 - What is the range of D(T) for the given domain?
Answer: For the domain T:[0,2000] , D(0) = 385,000 km and D(2000) = 461,000 km so the range is **D:[385,000 , 461,000]**

Problem 5 - How many years in the future will the orbit be exactly D(T) = 423,000 kilometers? Answer: solve 423,000 = 38T + 385,000

38T = 38,000

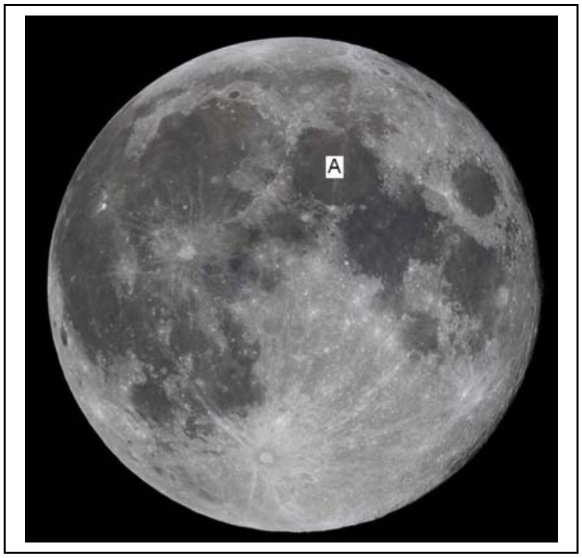
So T = 1,000

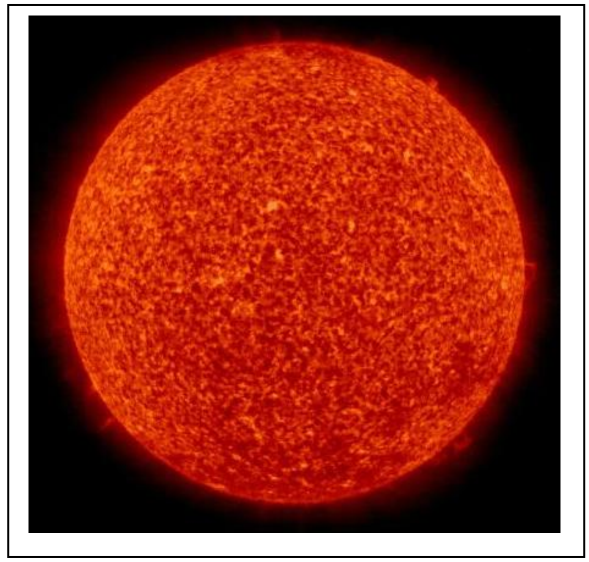
Since T is in units of millions of years, T = 1,000 is 1,000 million years or **1 billion years.**

Problem 6 - How far from Earth will the Moon be by 500 million years from now?
Answer: T = 500, so D(500) = 38(500) + 385,000 = **404,000 kilometers.**

Problem 7 - How far will the Moon be from Earth by T = 3,000?
Answer: **This value for T falls outside the stated domain of D(T) so we cannot use the function to determine an answer.**

Getting an Angle on the Sun and Moon





The Sun (Diameter = 1,392,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

Answer Key

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = **28.7 asec/mm** and the solar scale is 1865 asec/61 mm = **30.6 asec/mm.**

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or 0.5 x 28.7 asec/mm = **14.4 asec for the Moon** and 0.5 x 30.6 asec/mm = **15.3 asec for the Sun**.

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is $A = \pi \times (143.5 \text{ asec})^2 =$ **$64{,}700 \text{ asec}^2$**.

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to 400 x 1.9 km = **760 kilometers per arcsecond**.

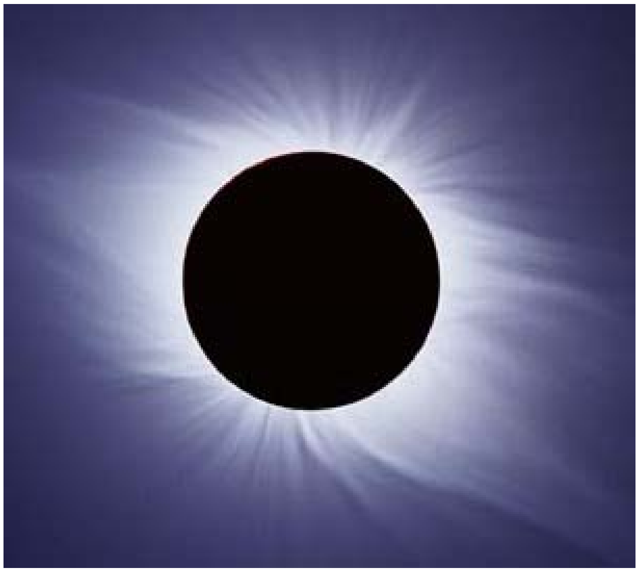
Problem 5 - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

$$64{,}700 \text{ asec}^2 \times (1.9 \text{ km/asec}) \times (1.9 \text{ km/asec}) = \mathbf{233{,}600 \text{ km}^2}.$$

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

$$64{,}700 \text{ asec}^2 \times (760 \text{ km/asec}) \times (760 \text{ km/asec}) = \mathbf{37{,}400{,}000{,}000 \text{ km}^2}.$$

The Last Total Solar Eclipse...Ever!



Total solar eclipses happen because the angular size of the moon is almost exactly the same as the sun's, despite their vastly different distances and sizes.

The moon has been steadily pulling away from earth over the span of billions of years. There will eventually come a time when these two angular sizes no longer match up. The moon will be too small to cause a total solar eclipse.

When will that happen?

Image courtesy Fred Espenak
http://sunearth.gsfc.nasa.gov/eclipse/eclipse.html

Problem 1 - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Problem 2 - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Problem 3 - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Problem 4 - How much further away from Earth will the moon be at that time?

Problem 5 - The moon is moving away from Earth at a rate of 3.8 centimeters per year. How many years will it take to move 3.8 kilometer further away?

Problem 6 - How many years will it take to move the distance from your answer to Problem 4?

Problem 7 - When will the last Total Solar Eclipse be sighted in the future?

Answer Key:

Problem 1 - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Answer: Because objects appear smaller the farther away they are, if you double the distance, the moon will appear half its former size, or 0.559/2 = 0.279 degrees across.

Problem 2 - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Answer: The distance is now 356,400 kilometers + 50,000 kilometers = 406,400 kilometers. The distance has increased by 406,400/356,400 = 1.14, so that means that the angular size has been reduced to 0.559 / 1.14 = 0.49 degrees.

Problem 3 - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Answer: 0.559/0.525 = 1.06 times further away from Earth or 356,400 km x 1.06 = 377,800 kilometers.

Problem 4 - How much further away from Earth will the moon be at that time?

Answer: 377,800 kilometers - 356,400 kilometers = 21,400 kilometers.

Problem 5: The moon is moving away from Earth at a rate of 3.8 centimeters per year. How many years will it take to move 3.8 kilometer further away?

Answer: (380,000 centimeters) / (3.8 centimeters / year) = 100,000 years.

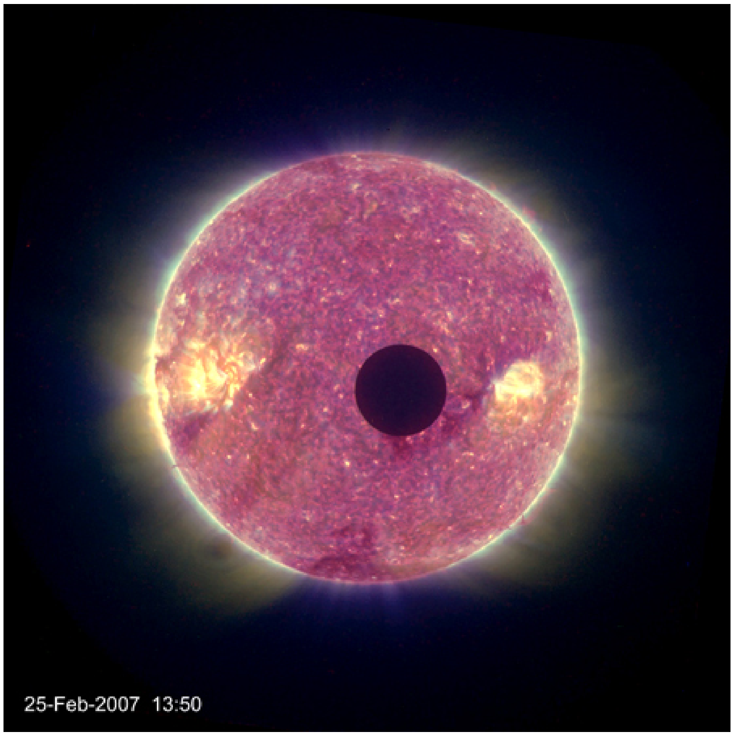
Problem 6: How many years will it take to move the distance from your answer to Problem 4?

Answer: (21,400 kilometers / 3.8 kilometers) x 100,000 years = 563 million years.

Problem 7: When will the last Total Solar Eclipse be sighted in the future?

Answer: About 563 million years from now.

A Lunar Transit of the Sun from Space



The twin STEREO satellites captured this picture of our Moon passing across the sun's disk on February 25, 2007. The two satellites are located approximately in the orbit of Earth, but are moving away from Earth in opposite directions. From this image, you can figure out how far away from the Moon the STEREO-B satellite was when it took this picture! To do this, all you need to know is the following:

1) The diameter of the Moon is 3,476 km

2) The distance to the Sun is 150 million km.

3) The diameter of the Sun is 0.54 degrees

Can you figure out how to do this using geometry?

Problem 1: Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the Sun was 0.54 degrees across on February 25. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Problem 2: As seen from the distance of Earth, the Moon has an Apparent Size of 0.53 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

Problem 3: From your answer to Problem 1, and Problem 2, what is the distance to the Moon from where the above photo was taken by the STEREO-B satellite?

Problem 4: On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters to the same scale!) with a compass, ruler and protractor?

Answer Key:

Problem 1: Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the sun is 0.5 degrees across. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Answer: The diameter of the sun is 57 millimeters. This represents 0.54 degrees, so the image scale is 0.54 degrees / 57 millimeters = 0.0095 degrees/mm
The diameter of the Moon is 12 millimeters, so the angular size of the Moon is

12 mm x 0.0095 degrees/mm = 0.11 degrees.

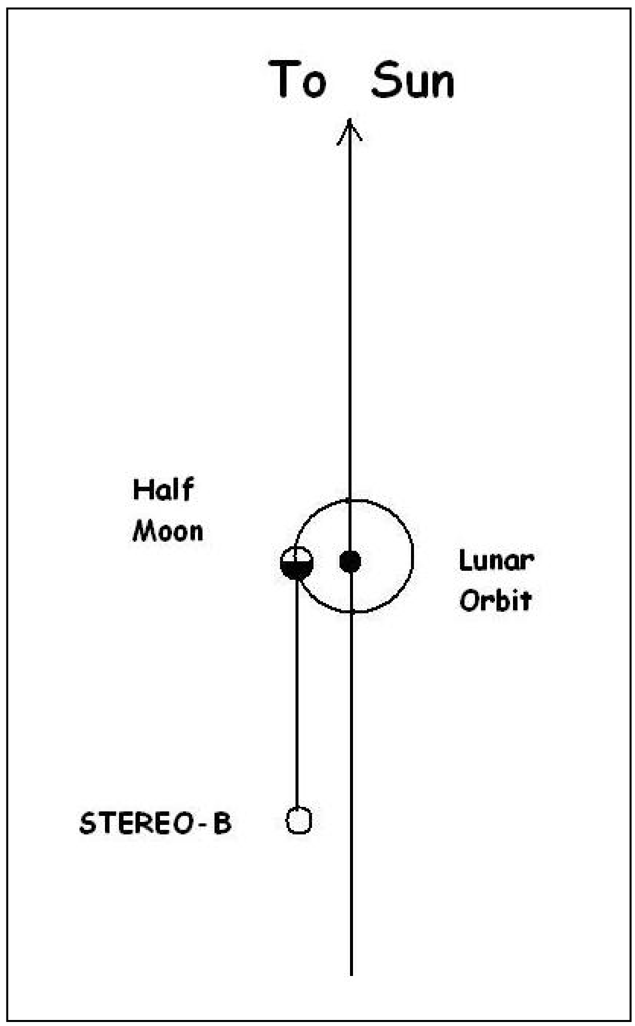
Problem 2: As seen from the distance of Earth, the Moon has an Apparent Size of 0.53 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?
Answer: It would have an Apparent Size half as large, or 0.26 degrees.

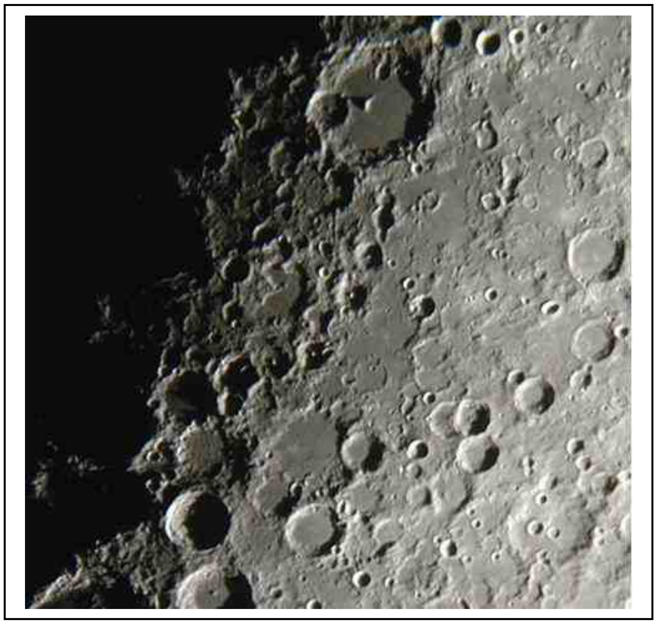
Problem 3: From your answer to Problem 1, and Problem 2, what is the distance to the moon from where the above photo was taken by the STEREO-B satellite?

Answer: The ratio of the solar diameter to the lunar diameter is 0.54 degrees/0.11 degrees = 4.9. This means that from the vantage point of STEREO, it is 4.9 times farther away than it would be at the Earth-Moon distance. This means it is 4.9 times farther away than 384,000 km, or 1.9 million kilometers.

Problem 4: On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters!) using a compass, ruler and protractor?

Answer: The figure to the right shows the locations of the Earth, Moon and STEREO satellite. The line connecting the Moon and the Satellite is 4.9 times the Earth-Moon distance.





One of the first things that astronomers wish to learn about a planet or other body in the solar system is the number of craters on its surface. This information can reveal, not only the age of the surface, but also the history of impacts during the age of the body.

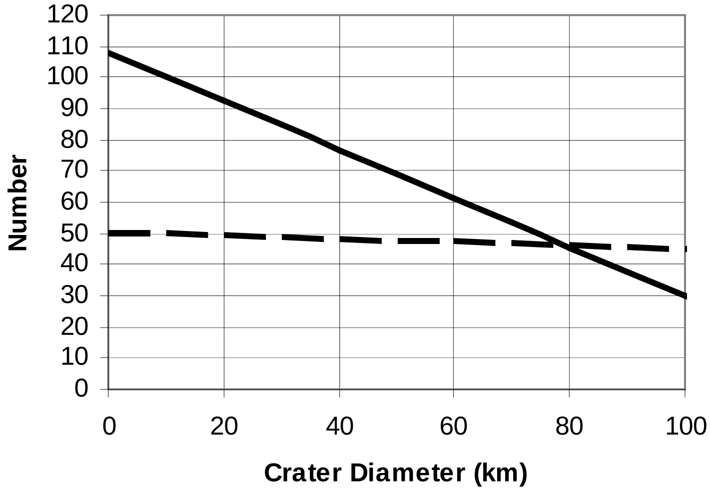
Bodies with no atmospheres preserve all impacts, regardless of size, while bodies with atmospheres or crustal activity, often have far fewer small craters compared to larger ones.

Studies of the number of craters on Venus and Mars have determined that for Venus, the number of craters with a diameter of D kilometers is approximated by $N = 108 - 0.78D$ while for Mars the crater counts can be represented by $N = 50 - 0.05 D$.

Problem 1 – Graphically solve these two equations to determine for what crater diameter the number of craters counted on the two planets is the same over the domain D:[0,100 km].

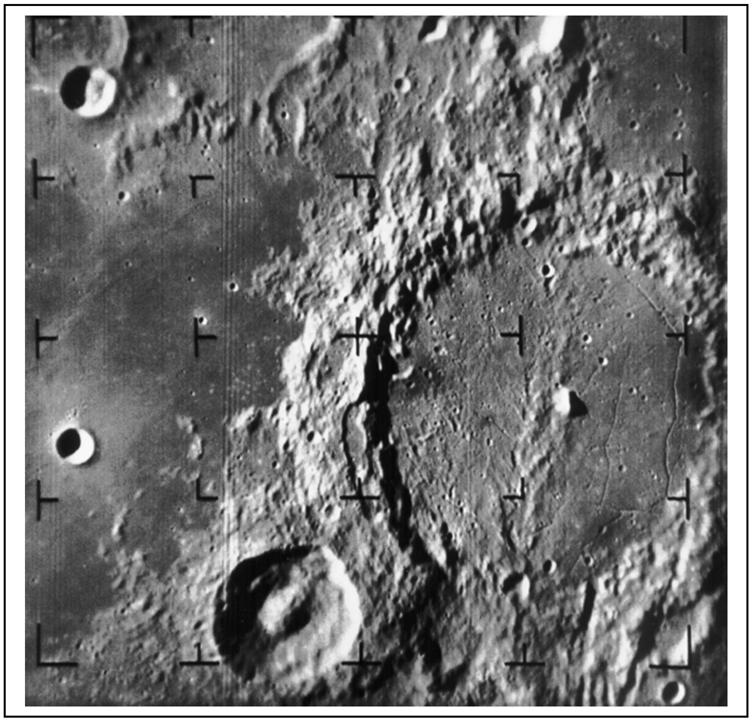
Problem 1 – Graphically solve these two equations to determine for what crater diameter the number of craters counted on the two planets is the same over the domain D:[0,100 km].

Answer:



Dashed line is the data for Mars, solid line is the data for Venus.
The intersection point is at D = 80 kilometers with N = 46 craters.

Craters are a Blast!



Have you ever wondered how much energy it takes to create a crater on the Moon. Physicists have worked on this problem for many years using simulations, and even measuring craters created during early hydrogen bomb tests in the 1950's and 1960's. One approximate result is a formula that looks like this:

$$\mathbf{E = 4.0 \times 10^{15}\ D^{3}\ Joules.}$$

where D is the crater diameter in kilometers.

As a reference point, nuclear bomb with a yield of one-megaton of TNT produces 4.0×10^{15} Joules of energy!

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified.

Note: To get a better sense of scale, the table below gives some equivalent energies for famous historical events:

Event	Equivalent Energy (TNT)
Cretaceous Impactor	100,000,000,000 megatons
Valdiva Volcano, Chile 1960	178,000 megatons
San Francisco Earthquake 1909	600 megatons
Hurricane Katrina 2005	300 megatons
Krakatoa Volcano 1883	200 megatons
Tsunami 2004	100 megatons
Mount St. Helens Volcano 1980	25 megatons

Answer Key

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

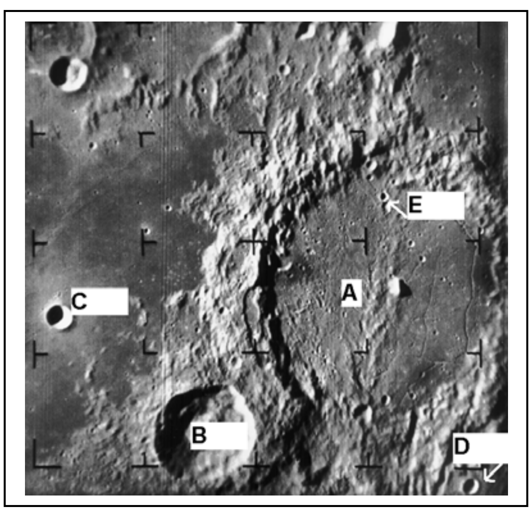
Answer: $E = 4.0 \times 10^{15} D^3$ Joules x (1 megaton TNT/4.0×10^{15} Joules)

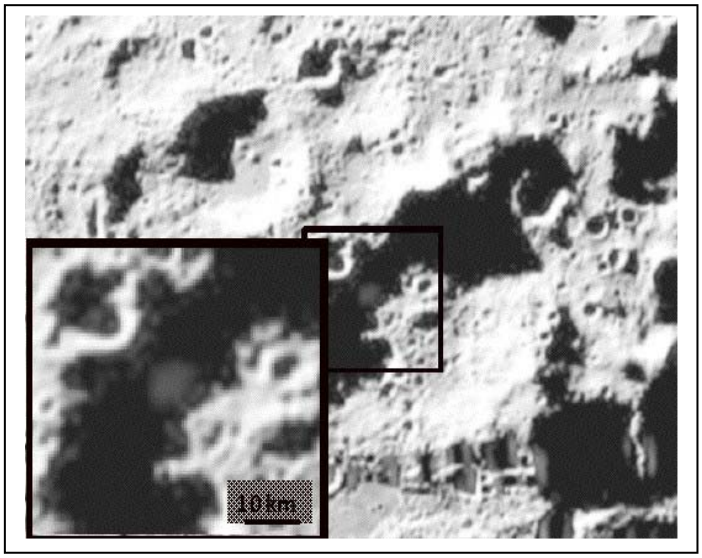
$$E = 1.0 \ D^3 \text{ Megatons of TNT}$$

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture. Answer: The width of the image is 92 mm, so the scale is 183/92 = 2.0 km/mm. See figure below for some typical examples: See column 3 in the table below for actual crater diameters.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified. Answer: See the table below, column 4. Crater A is called Alphonsis. Note: No single formula works for all possible scales and conditions. The impact energy formula only provides an estimate for lunar impact energy because it was originally designed to work for terrestrial impact craters created under Earth's gravity and bedrock conditions. Lunar gravity and bedrock conditions are somewhat different and lead to different energy estimates. The formula will not work for laboratory experiments such as dropping pebbles onto sand or flour. The formula is also likely to be inaccurate for very small craters less than 10 meters, or very large craters greatly exceeding the sizes created by nuclear weapons. (e.g. 1 kilometer).

Crater	Size (mm)	Diameter (km)	Energy (Megatons)
A	50	**100**	**1,000,000**
B	20	**40**	**64,000**
C	5	**10**	**1,000**
D	3	**6**	**216**
E	1	**2**	**8**





On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If density = mass/volume, and the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket, what was the concentration of water in the plume in units of tons of regolith per liter of water?

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Answer: $V = (3.14) \times (10 \text{ meters})^2 \times 3$ meters = **942 cubic meters**.

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Answer: 3000 kg/m^3 x (942 meters^3) = 2,800,000 kilograms. Since 1000 kg = 1 ton, there were **2,800 tons of regolith excavated**.

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

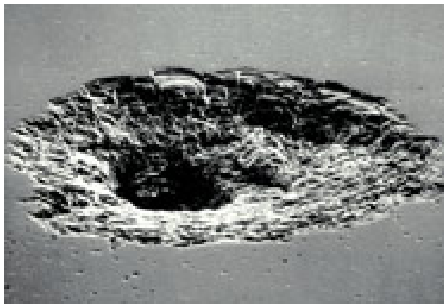
Answer: The area of the ejecta blanket is given by $A = \pi(35 \text{ meters})^2 - \pi(10 \text{ meters})^2$ = 3,846 - 314 = 3500 meters^2. The volume is A x h = (3500 meters^2) x 0.2 meters = 700 meters^3 . Then the mass is just M = (700 meters^3) x (3,000 kilograms/meter^3) = 2,100,000 kilograms or **2,100 tons in the ejecta blanket**.

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Answer: The amount of ejected regolith was 2,800 tons - 2,100 tons or 700 tons. The detected water amounted to 25 gallons or 25 gallons x (3.78 liters/ 1 gallon) = 95 liters. So the concentration was about C = 700 tons/95 liters = **7 tons/liter**.

Note to teacher: The estimated concentration, C, in Problem 4 is based on an approximated geometry for the crater (cylinder), an average thickness for the ejecta blanket (about 0.2 meters) and whether all of the remaining material (700 tons) was actually involved in the plume measured by LCROSS. Students may select, by scaled drawing, other geometries for the crater, and thickness for the ejecta blanket to obtain other estimates for the concentration, C. The scientific analysis of the LCROSS data may eventually lead to better estimates for C.

Is There a Lunar Meteorite Hazard?



Damage to Space Shuttle Endeavor in 2000 from a micrometeoroid or debris impact . The crater is about 1mm across. (Courtesy - JPL/NASA)

Without an atmosphere, there is nothing to prevent millions of pounds a year of rock and ice fragments from raining down upon the lunar surface.

Traveling at 10,000 miles per hour (19 km/s), they are faster than a speeding bullet and are utterly silent and invisible until they strike.

Is this something that lunar explorers need to worry about?

Problem 1 - Between 1972 and 1992, military infra-sound sensors on Earth detected 136 atmospheric detonations caused by meteors releasing blasts carrying an equivalent energy of nearly 1,000 tons of TNT - similar to small atomic bombs, but without the radiation. Because many were missed, the actual rates could be 10 times higher. If the radius of Earth is 6,378 km, A) what is the rate of these deadly impacts on Earth in terms of impacts per km^2 per year? B) Assuming that the impact rates are the same for Earth and the Moon, suppose a lunar colony has an area of 10 km^2. How many years would they have to wait between meteor impacts?

Problem 2 - Between 2005-2007, NASA astronomers counted 100 flashes of light from meteorites striking the lunar surface - each equivalent to as much as 100 pounds of TNT. If the surveyed area equaled 1/4 of the surface area of the Moon, and the lunar radius is 1,737 km, A) What is the arrival rate of these meteorites in meteorites per km^2 per year? B) If a lunar colony has an area of 10 km^2, how long on average would it be between impacts?

Problem 3 - According to H.J. Melosh (1981) meteoroids as small as 1-millimeter impact a body with a 100-km radius about once every 2 seconds. A) What is the impact rate in units of impacts per m^2 per hour? B) If an astronaut spent a cumulative 1000 hours moon walking and had a spacesuit surface area of 10 m^2, how many of these deadly impacts would he receive? C) How would you interpret your answer to B)?

Answer Key

Problem 1 - A) The surface area of Earth is $4 \pi (6378)^2 = 5.1 \times 10^8$ km^2. The rate is R = 136 x 10 impacts / 20 years / 5.1×10^8 km^2 = **1.3×10^{-7} impacts/km^2/year.**

B) The number of impacts/year would be 1.3×10^{-7} impacts/km^2/year x 10 km^2 = 1.3×10^{-6} impacts/year . The time between impacts would be $1/1.3 \times 10^{-6}$ = **769,000 years!**

Problem 2 - A) The total surface area of the Moon is $4 \pi (1737)^2 = 3.8 \times 10^7$ km^2. Only 1/4 of this is surveyed so the area is 9.5×10^5 km^2. Since 100 were spotted in 2 years, the arrival rate is R = 100 impacts/2 years/ 9.5×10^5 km^2 = **5.3×10^{-5} impacts/km^2/year**.

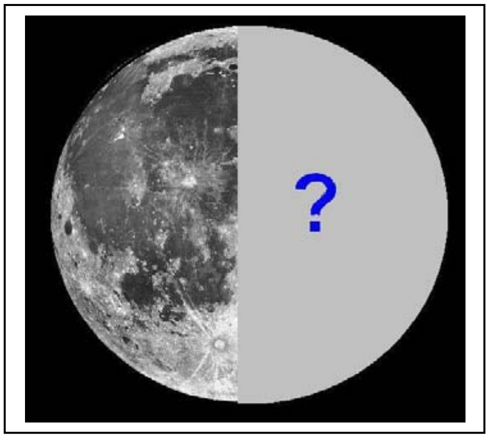
B) The rate for this area is 10 km^2 x 5.3×10^{-5} impacts/km^2/year = 5.3×10^{-4} impacts/year, so the time between impacts is about 1/ 5.3×10^{-4} = **1,900 years**

Problem 3 - A) A sphere 100-km in radius has a surface area of $4 \pi (100{,}000)^2 = 1.3 \times 10^{11}$ m^2. The impacts arrive every 2 seconds on average, which is 2/3600 = 5.6×10^{-4} hours. The rate is, therefore, R = 1 impacts / (1.3×10^{11} m^2 X 5.6×10^{-4} hours) = **1.4×10^{-8} impacts/m^2/hour.**

B) The number of impacts would be 1.4×10^{-8} impacts/m^2/hour x 10 m^2 x 1000 hours = **1.4×10^{-5} impacts.**

C) Because the number of impacts is vastly less than 1 (a certainty), he should not worry about such deadly impacts unless he had reason to suspect that the scientists miscalculated the impact rates for meteorites this small. Another way to look at this low number is to turn it around and say that the astronaut would have to take 1/ 1.4×10^{-5} about 71,000 such 1000-hour moon walks in order for one impact to occur. Alternately, the time between such events is 71,000 x 1000 hours = 71 million years!

The Moon's Density - What's inside?



The Moon has a mass of 7.4 x 10^{22} kilograms and a radius of 1,737 kilometers. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 kilometers where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior.

Problem 1 - What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

Problem 2 - What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

Problem 3 - From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

Problem 4 - The densities of various rock types are given in the table below.

Type	Density
I - Iron+Nickle mixture (Earth's core)	15.0 gm/cc
E - Earth's mantle rock (compressed)	4.5 gm/cc
B - Basalts	2.9 gm/cc
G - Granite	2.7 gm/cc
S - Sandstone	2.5 gm/cc

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4 x 10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Problem 1 - Mass = 7.4×10^{22} kg x 1000 gm/kg = 7.4×10^{25} grams. Radius = 1,737 km x 100,000 cm/km = 1.737×10^{8} cm. Volume of a sphere = $4/3 \pi R^3 = 4/3 \times (3.141) \times (1.737 \times 10^8)^3 = 2.2 \times 10^{25}$ cm^3, so the density = 7.4×10^{25} grams / 2.2×10^{25} cm^3 = **3.4 gm / cm^3.**

Problem 2 - A) V(core) = $4/3 \pi R^3 = 4/3 \times (3.141) \times (4.0 \times 10^7)^3$ = **2.7×10^{23} cm^3**
B) V(shell) = V(Rsurface) - V(Rcore) = 2.2×10^{25} cm^3 - 2.7×10^{23} cm^3 = **2.17×10^{25} cm^3**

Problem 3 - The total core mass is given by M(core) = $4/3 \pi (Rcore)^3 \times D1$. The volume of the mantle shell is given by multiplying the shell volume V(shell) calculated in Problem 2B by the density : Mshell = V(shell) x D2. Then, the formula for the total mass of the model is given by: $MT = 4/3 \pi (Rc)^3 \times D1 + (4/3 \pi (Rs)^3 - 4/3 \pi (Rc)^3) \times D2$, which can be simplified to:

$$MT = 4/3 \pi \, (D1 \times Rc^3 + D2 \times Rs^3 - D2 \times Rc^3)$$

Problem 4 - A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5

E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7×10^{25} and 6.4×10^{25} grams respectively. These are both very close to the actual moon mass of 7.4×10^{25} grams (e.g. 7.4×10^{22} kilograms) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron/nickel mixture (15 gm/cc).

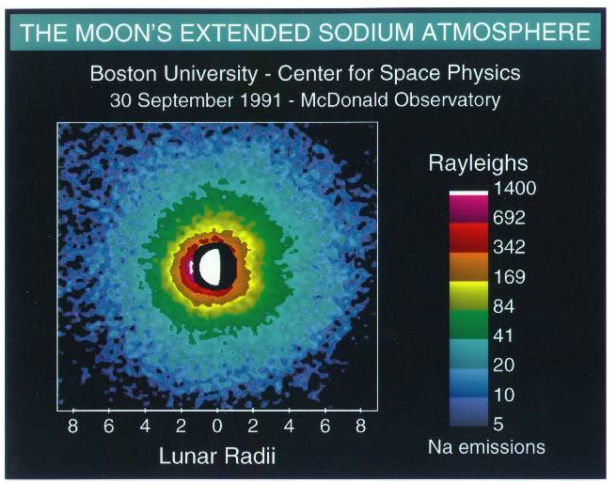
Investigating the Lunar Atmosphere

Courtesy: T.A.Rector, I.P.Dell'Antonio (NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6×10^{-24} grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

Answer Key:

Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6 x 10-24 grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen = 1.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 6.4 x $10^{\mathbf{-20}}$ grams
B) Helium = 4.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.6 x $10^{\mathbf{-19}}$ grams
C) Neon = 20.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 1.3 x $10^{\mathbf{-18}}$ grams
D) Argon = 36.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.3 x $10^{\mathbf{-18}}$ grams
E) Total = (0.064 + 0.26 + 1.3 + 2.3) x 10^{-18} grams = 3.9 x $10^{\mathbf{-18}}$ grams per cc.

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of Ri = 1,738 km and Ro = 1,738+170 = 1,908 km. V = 4/3 π $(1908)^{\mathbf{3}}$ - 4/3π $(1738)^{\mathbf{3}}$ = 2.909 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ - 2.198 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$

Volume = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ **x** (10^5 cm/km) **x** (10^5 cm/km) **x** (10^5 cm/km)
= 7.1 x 10^{24} cm^3

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of Ri, multiplied by the shell thickness of h = 170 km, you will get a slightly different answer of 6.5 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ and 6.5 x 10^{24} cm^3

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

A) Mass = density x volume = (3.9 x $10^{\mathbf{-18}}$ gm/cc) x 7.1 x $10^{\mathbf{24}}$ $cm^{\mathbf{3}}$ = 2.8 x $10^{\mathbf{7}}$ grams

B) Mass = 2.8 x $10^{\mathbf{7}}$ grams x (1 kg/1000 gms) = 28,000 kilograms.

C) Mass = 28,000 kg x (1 ton / 1000 kg) = 28 tons.

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.

Extracting Oxygen from Moon Rocks

About 85% of the mass of a rocket is taken up by oxygen for the fuel, and for astronaut life support. Thanks to the Apollo Program, we know that as much as 45% of the mass of lunar soil compounds consists of oxygen. The first job for lunar colonists will be to 'crack' lunar rock compounds to mine oxygen.

NASA has promised $250,000 for the first team capable of pulling breathable oxygen from mock moon dirt; the latest award in the space agency's Centennial Challenges program.

Lunar soil is rich in oxides of silicon, calcium and iron. In fact, 43% of the mass of lunar soil is oxygen. One of the most common lunar minerals is *ilmenite*, a mixture of iron, titanium, and oxygen. To separate *ilmenite* into its primary constituents, we add hydrogen and heat the mixture. This hydrogen reduction reaction is given by the 'molar' equation:

$$FeTiO_3 + H_2 \; \text{--->} \; Fe + TiO_2 + H_2O$$

A Bit Of Chemistry - This equation is read from left to right as follows: One mole of *ilmenite* is combined with one mole of molecular hydrogen gas to produce one mole of free iron, one mole of titanium dioxide, and one mole of water. Note that the three atoms of oxygen on the left side (O_3) is 'balanced' by the three atoms of oxygen found on the right side (two in TiO_2 and one in H_2O). **One 'mole' equals 6.02×10^{23} molecules**.

The 'molar mass' of a molecule is the mass that the molecule has if there are 1 mole of them present. The masses of each atom that comprise the molecules are added up to get the molar mass of the molecule. Here's how you do this:

For H_2O, there are two atoms of hydrogen and one atom of oxygen. The atomic mass of hydrogen is 1.0 AMU and oxygen is 16.0 AMU, so the molar mass of H_2O is 2 (1.0) + 16.0 = 18.0 AMU. **One mole of water molecules will equal 18 grams of water by mass.**

Problem 1 -The atomic masses of the atoms in the *ilmenite* reduction equation are Fe = 55.8 and Ti = 47.9. A) What is the molar mass of ilmenite? B) What is the molar mass of molecular hydrogen gas? C) What is the molar mass of free iron? D) What is the molar mass of titanium dioxide? E) Is mass conserved in this reaction?

Problem 2 - If 1 kilogram of ilmenite was 'cracked' how many grams of water would be produced?

Inquiry Question - If 1 kilogram of ilmenite was 'cracked' how many grams of molecular oxygen would be produced if the water molecules were split by electrolysis into

$$2\,H_2O \; \text{-->} \; 2H_2 + O_2?$$

Problem 1 -

A) What is the molar mass of ilmenite? 1(55.8) + 1 (47.9) + 3(16.0) = **151.7 grams/mole**

B) What is the molar mass of molecular hydrogen gas? 2(1.0) = **2.0 grams/mole**

C) What is the molar mass of free iron? 1(55.8) = **55.8 grams/mole**

D) What is the molar mass of titanium dioxide? 1(47.9) + 2(16.0) = **79.9 grams/mole**

E) Is mass conserved in this reaction? **Yes. There is one mole for each item on each side, so we just add the molar masses for each constituent. The left side has 151.7 + 2.0 = 153.7 grams and the right side has 55.8 + 79.9 + 18.0 = 153.7 grams so the mass balances on each side.**

Problem 2 -

Step 1 - The reaction equation is balanced in terms of one mole of ilmenite (1.0 x $FeTiO_3$) yielding one mole of water (1.0 x H_2O) . The molar mass of ilmenite is 151.7 grams which is the same as 0.1517 kilograms, so we just need to figure out how many moles is needed to make one kilogram.

Step 2 - This will be 1000 grams/151.7 grams = 6.6 moles. Because our new reaction is that we start with 6.6 x $FeTiO_3$ that means that for the reaction to remain balanced, we need to produce 6.6 x H_2O, or in other words, 6.6 moles of water.

Step 3 - Because the molar mass of water is 18.0 grams/mole, the total mass of water produced will be 6.6 x 18.0 = **119 grams of water.**

Inquiry Question - The reaction is: $2\ H_2O \rightarrow 2H_2 + O_2$

This means that for every 2 moles of water, we will get one mole of O_2. The ratio is 2 to 1. From the answer to Problem 2 ,we began with 6.59 moles of water not 2.0 moles. That means we will produce 6.6/2 = 3.3 moles of water. Since 1 molecule of oxygen has a molar mass of 2(16) = 32 grams/mole, the total mass of molecular oxygen will be 3.3 moles x 32 grams/mole = **106 grams. So, 1 kilogram of ilmenite will eventually yield 106 grams of breathable oxygen.**

The Mass of the Moon

On July 19, 1969 the Apollo-11 Command Service Module and LEM entered lunar orbit. The orbit period was 2.0 hours, at a distance of 1,737 kilometers from the lunar center.

Believe it or not, you can use these two pieces of information to determine the mass of the moon. Here's how it's done!

Problem 1 - Assume that Apollo-11 went into a circular orbit, and that the inward gravitational acceleration by the moon on the capsule, Fg, exactly balances the outward centrifugal acceleration, Fc. Solve Fc = Fg for the mass of the moon, M, in terms of V, R and the constant of gravity, G, given that:

$$F_g = \frac{G M m}{R^2} \qquad F_c = \frac{m V^2}{R}$$

Problem 2 - By using the fact that for circular motion, $V = 2 \pi R / T$, re-express your answer to Problem 1 in terms of R, T and M.

Problem 3 - Given that $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}$, R = 1,737 kilometers and T = 2 hours, calculate the mass of the moon in kilograms!

Problem 4 - The mass of Earth is 5.97×10^{24} kilograms. What is the ratio of the moon's mass, derived in Problem 3, to Earth's mass?

Answer Key

Problem 1 - From Fg = Fc, and a little algebra to simplify and cancel terms, you get

$$M = \frac{R V^2}{G}$$

Problem 2 - Substitute 2 π R / T for V and with little algebra to simplify and cancel terms, you get :

$$M = \frac{4\pi^2 R^3}{G T^2}$$

Problem 3 - First convert all units to meters and seconds: R = 1.737×10^6 meters and T = 7,200 seconds. Then substitute values into the above equation:

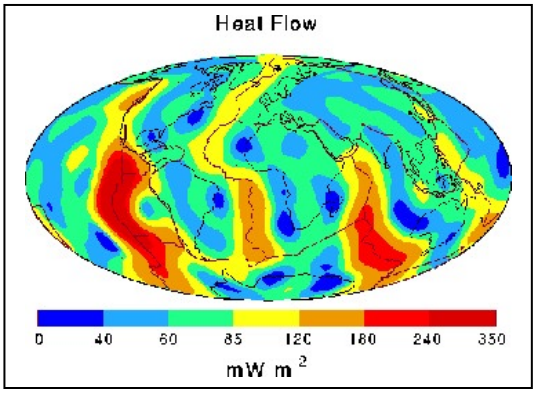
$M = 4 \times (3.14)^2 \times (1.737 \times 10^6)^3 / (6.67 \times 10^{-11} \times (7200)^2))$

$M = (39.44 \times 5.24 \times 10^{18}) / (3.46 \times 10^{-3})$

M = **5.97×10^{22} kilograms**

More accurate measurements, allowing for the influence of Earth's gravity and careful timing of orbital periods, actually yield 7.4×10^{22} kilograms.

Problem 4 - The ratio of the masses is 5.97×10^{22} kilograms / 5.97×10^{24} kilograms which equals **1/100.** The actual mass ratio is 1 / 80.



Earth heat flow map (H. N. Pollack, S. J. Hurter, and J. R. Johnson, Reviews of Geophysics, Vol. 31, 1993.)

When large bodies form, their interiors are heated by a combination of radioactivity and the heat of formation from the infall of the rock. For planet-sized bodies, this heat can be generated and lost over billions of years. The end result will be that the core cools off and, if molten, it eventually solidifies. Measuring the surface temperature of a solid body, and the rate at which heat escapes its surface, provides clues to its internal heating and cooling rates.

Problem 1 - Measuring the heat flow out of the lunar surface is a challenge because the monthly and annual changes of surface solar heating produce interference. Apollo 15 astronauts measured the heat flow from two bore holes that reached about 2-meters below the surface. When corrected for the monthly effects from the Sun, they detected a heat flow of about 20 milliWatts/meter^2. If the radius of the Moon is 1,737 kilometers, what is the total thermal power emitted by the entire Moon in billions of watts?

Problem 2 - A future lunar colony covers a square surface that is 100 meters x 100 meters. What is the total thermal power available to this colony by 'harvesting' the lunar heat flow?

Problem 3 - The relationship between power, L, surface radius, R, and surface temperature, T, is given by $L = 4 \pi R^2 \sigma T^4$ where σ = the Stefan-Boltzman constant and has a value of 5.67×10^{-8} W/m^2 / K^4 , and where T is in Kelvin degrees, L is in watts, and R is in meters. Suppose the Moon's interior was heated by a source with a radius of 400 kilometers at the lunar core, what would the temperature of this core region have to be to generate the observed thermal wattage at the surface?

Problem 4 - The lunar regolith and crust is a very good insulator! Through various studies, the temperature of the Moon is actually believed to be near 1,200 K within 400 km of the center. A) Using the formula for L in Problem 3, how much power is absorbed by the lunar rock overlaying the core? B) From you answer to (A), how many joules are absorbed by each cubic centimeter of overlaying lunar rock each second (joules/cm$^{\mathbf{3}}$)? C) Basalt begins to soften when in absorbs over 1 million Joules/cm^3. Is the lunar surface in danger of melting from the heat flow within?

Problem 1 - The surface area of a sphere is 4 π R^2, so the surface area of the moon is 4 x 3.14 x $(1.737 \times 10^6)^2$ = 3.8 x 10^{18} m^2 .The total power, in watts, is then 0.020 watts/m^2 x 3.8 x 10^{18} m^2 = 7.6 x 10^{11} watts or **760 billion watts**.

Problem 2 - The surface area is 100 x 100 = 10,000 m^2, and with a heat flow of 0.020 milliwatts/m^2, the total thermal power is **200 watts.**

Problem 3 - The total thermal power, L = 7.6 x 10^{11} watts, and R = 400 km = 4.0 x 105 meters, so that 7.6 x 10^{11} = 4 x (3.14) x $(4.0 \times 10^5)^2$ x 5.67×10^{-8} T^4
Then 7.6 x 10^{11} = 1.1 x 10^5 T^4. Solving for T we get **T = 51 K**.

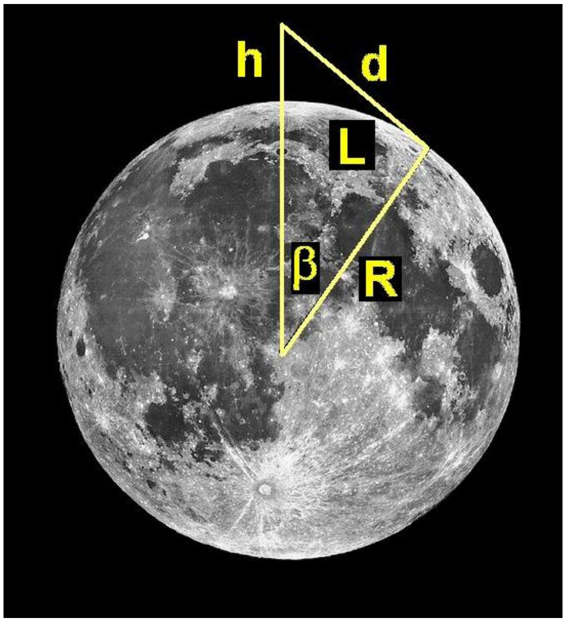
Problem 4 - A) The power emitted by the 400 km, 1,200 K core region is given by

$$L = 4 \pi R^2 \sigma T^4$$

which equals L = 4 x (3.14) x $(4.0 \times 10^5)^2$ x 5.67×10^{-8} $(1200)^4$ = 2.4 x 10^{17} watts. Since from the answer to Problem 1 the amount that makes it to the surface is only 7.6 x 10^{11} watts, that leaves essentially all of the 2.4 x 10^{17} watts to be absorbed by the overlaying rock mantle.

B) The volume of overlaying rock is the total volume of the moon (radius 1,737 km) minus the volume of the 400-km lunar core. The difference in these two spherical regions is: 4/3 π ($(1.737 \times 10^6)^3$ - $(4.0 \times 10^5)^3$) = 4/3 x 3.14 x (5.18 x 10^{18}) = 2.2 x 10^{19} m^3, or 2.2 x 10^{25} cm^3. Since 1 watt = 1 Joule/sec, then in 1 second the lunar thermal power from the 1,200 K core is 2.4 x 10^{17} joules as calculated in (A). If this is evenly absorbed by the rock in the mantle, the average thermal heating energy per cm^3 is just 2.4 x 10^{17} joules / 2.2 x 10^{25} cm^3. or **1.0 x 10^{-6} joules/cm^3. (This is also equal to 10 ergs /cm^3)**

C) **No, because this amount of energy input is completely negligible in melting, or warming, rock material.** Note: Basalt softens at 1,200 C = 1,500 K. A cubic centimeter of this rock has a surface area of 6 cm^2, so from L = SA x σ T^4 we get L = 6.0 x 5.67×10^{-8} $(1500)^4$ = 1.7 x 10^6 watts, which in 1 second amounts to 1.7 x 10^6 Joules / cm^3 - the energy needed to melt basalt.



An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna on the Moon in order to insure proper reception out to a specified distance.

Teachers: *Problems 1-4 can be successfully accomplished by algebra students. Problems 5 and 6 require a knowledge of derivatives and can be assigned to calculus students after they have completed Problem 1 and 2.*

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure above to derive the formula for the line-of-sight (LOS) horizon distance, d, to the horizon tangent point.

Problem 2: Derive the distance along the planet, L, to the tangent point.

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) The Moon (1,738 km)

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance in the Moon?

Problem 5: What is the rate of change of the lunar LOS radius, d, for each additional meter of antenna height in Problem 4?

Problem 6: What is the rate-of-change of the distance to the lunar radio tower, L, at the LOS position in Problem 4?

Answer Key:

Problem 1: By the Pythagorean Theorem $d^2 = (R+h)^2 - R^2$

so $d = (R^2 + 2Rh + h^2 - R^2)^{1/2}$

and so $d = (h^2 + 2Rh)^{1/2}$

Problem 2: Derive the distance along the planet, l, to the tangent point. From the diagram,

$\cos(\beta) = R/(R+h)$ and so $L = R \arccos(R/(R+h))$

Problem 3: Use the equation from Problem 1. A) For Earth, R=6378 km and h=2 meters so $d = ((2 \text{ meters})^2 + 2 \times 2 \text{ meters} \times 6.378 \times 10^6 \text{ meters})^{1/2}$ = 5051 meters or 5.1 kilometers.
B) For the Moon, R=1,738 km so d = 2.6 kilometers

Problem 4: h = 50 meters, R=1,738 km so d = 13,183 meters or 13.2 kilometers.

Problem 5: Use the chain rule to take the derivative with respect to h of the equation for d in Problem 1. Evaluate dd/dh at h=50 meters for R=1,738 km.

Let $U = h^2 + 2Rh$ then $d = U^{1/2}$ so $dU/dh = (dd/dU)(dU/dh)$

Then $dd/dh = +1/2\, U^{-1/2}$

$dU/dh = +1/2\,(2h + 2R)(h^2 + 2Rh)^{-1/2}$

For h=50 meters and R = 1,738 km,

$dd/dh = +0.5 \times (100 + 3476000)(2500 + 2 \times 50 \times 1738000)^{-1/2}$

= **+131.8 meters in LOS distance per meter of height.**

Problem 6: Let $U = R/(R+h)$, then $L = R \cos^{-1}(U)$.

By the chain rule $dL/dh = (dL/dU) \times (dU/dh)$.

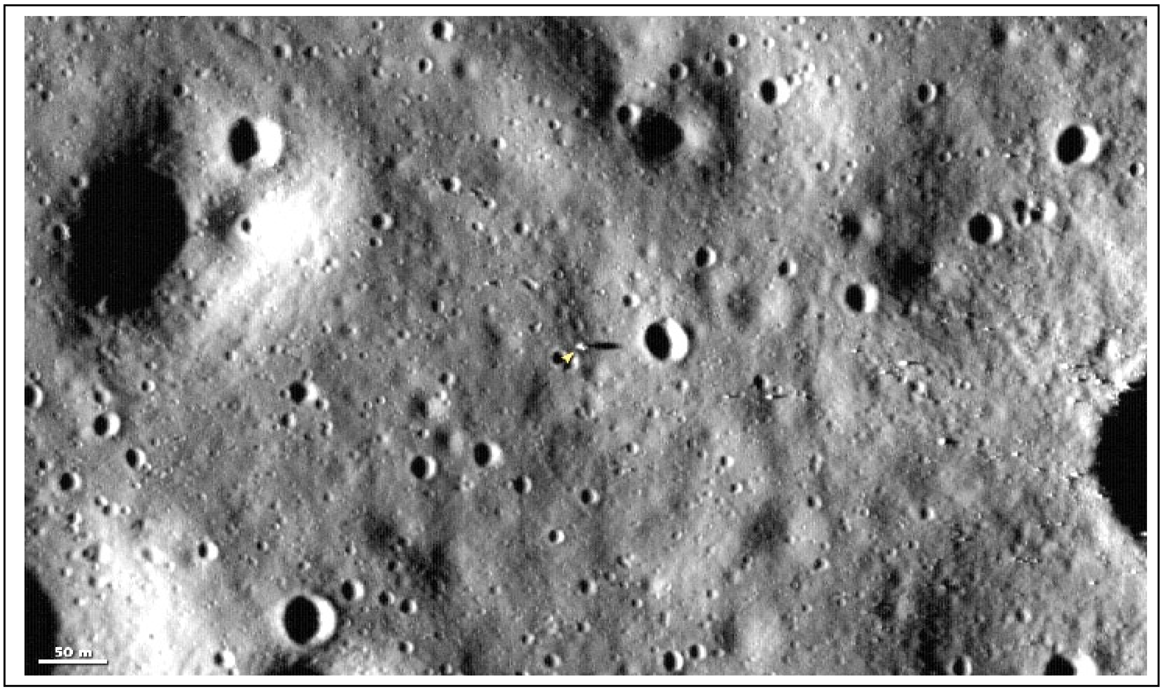
Since $dL/dU = R \times (-1)(1 - u2)^{1/2}$ and $dU/dh = R \times (-1) \times (R + h)^{-2}$ then

$dL/dh = R^2 (R + h)^{-2} (R + h)^{1/2} /((R+ h)^2 - R^2)^{1/2}$

$dL/dh = R^2 (R+h)^{-1} (h^2 + 2Rh)^{-1/2}$

Since R >> h, $\mathbf{dL/dh = R/(2Rh)^{1/2}}$

Evaluating this for R = 1,738 km and h = 50 meters gives **dL/dh = +131.8 meters per kilometer.**



This image of the 800-meter x 480-meter region near the Apollo-11 landing pad was taken by the Lunar Reconnaissance Orbiter (LRO). It reveals hundreds of craters covering the landing area with sizes as small as 5 meters. The Apollo-11 landing pad is at the center of the image, and is casting a long horizontal shadow to the right of the pad, in the direction of a small crater.

Astronomers use counts of the number of craters per kilometer^2 as a function of crater diameter to determine the age of a given lunar landscape, and the distribution of the sizes of the impactors. Crater counts are also used to determine which areas are safe to land. The power-law function below is based upon the above image from LRO and gives the surface density of craters near the Apollo-11 landing site in terms of craters per kilometer^2 of a given diameter, x, in meters. The range of validity is from 2 meters to 40 meters for this particular lunar area. Apollo-11 astronauts did not find any craters smaller than 2-meters near the landing area.

$$S(x) = 22000\, x^{-2.4} \quad \text{craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters.

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters.

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? (Assume that the craters do not overlap, which is a good approximation to what the image shows.)

$$S(x) = 22000\, x^{-2.4} \text{ craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters. Answer: The limits to the definite integral extend from m to infinity:

$$\int_m^{\infty} 22000 x^{-2.4} dx \quad = \quad N(x>m) = \frac{22000}{1.4m^{1.4}}$$

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters. Answer: The limits to the definite integral extend from 2 to m, because Apollo-11 astronauts did not see any craters smaller than 2 meters (x=2) in this area:

$$\int_2^{m} 22000 x^{-2.4} dx \qquad N = \frac{22000}{1.4(2)^{1.4}} - \frac{22000}{1.4m^{1.4}} \quad \text{so} \quad N = 5955 - \frac{22000}{1.4m^{1.4}}$$

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? Answer: S(x) gives the number of craters with a diameter of x per km^2. The maximum area occupied by these craters assuming that they are non-overlapping is given by $\pi (x/2)^2$ S(x). The total area covered by non-overlapping craters larger than 2 meters is given by the integral:

$$A = \int_2^{40} \pi \left(\frac{x}{2}\right)^2 22000 x^{-2.4} dx$$

so $$A = \frac{22000\pi}{4} \int_2^{40} x^{-0.4} dx$$ then $$A = \frac{22000\pi}{4} \left(\frac{1}{0.6}\right) \left[x^{0.6} \right]_2^{40}$$

$$A = \frac{22000\pi}{4(0.6)} \left[40^{0.6} - 2^{0.6} \right]$$

so that the cratered area is A = 28783 (9.14 - 1.52) = 220,000 square meters. The area in the image is 800 meters x 480 meters = 384,000 square meters, so the cratered area represents 100% x (220,000/384,000) = 57% of the surface area. So, the **maximum area that is covered by craters is 57%.** Note: That means that 43% of the area was safe to land on.

Additional Math Resources about the Moon

There are many web-related resources that cover the moon from a mathematical perspective. Here are just a few…

Lunar Math Lander – An interactive game in which you try to land a spacecraft by answering math questions without crashing the spacecraft on the moon.
http://www.kidsnumbers.com/division-moon-math.php

NASA-QUEST: Moon Math Guide – Cratering! – A complete hands-on guide to crater mathematics, and calculating irregular areas.
http://lcross.arc.nasa.gov/docs/MM_Suppl_Guide_v1.pdf

Moon Math Challenge Guide (ESA/NASA) – An inquiry-based guide to a variety of moon projects for grades 5-8 that involve students asking questions based on measurements thay can make.
http://esc.nasa.gov/documents/MoonMathChallengeGuide.pdf

PUMAS – Can an Astronaut on Mars Distinguish Earth from the Moon? Could the unaided eye of an observer on Mars tell apart the Earth and its moon, at their greatest separation?
https://pumas.gsfc.nasa.gov/examples/index.php?id=7

PUMAS – The Cause of the Phases of the Moon –This activity is frequently used to show the causes of the lunar phases. Generally a teacher tells students how to position the ball to show a full moon, a crescent moon, etc.
https://pumas.gsfc.nasa.gov/examples/index.php?id=83

PUMAS – The Moon Orbits the Sun? –Their observations, or what is "common knowledge", lead them to believe the Moon does loops around the Earth. But is this true? A comparison of the gravitational forces of the Sun and Earth on the Moon hints at the answer to this question and a simple demonstration refutes the loop-view.
https://pumas.gsfc.nasa.gov/examples/index.php?id=86

The NASA Lunar Ephemeris – Gives distance to the moon for every day from the years 1995-2006. Can be used to explore lunar orbit shape, phases.
http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html

Interactive Lunar Ephemeris Calculator – calculates many lunar parameters for any day/year that you enter.
http://www.lunar-occultations.com/rlo/ephemeris.htm

Lunar Cratering Rates – How scientists calculate the rate of cratering on the lunar surface. Includes a graph showing crater size versus frequency.
http://www.psi.edu/projects/mgs/cratering.html

A note from the Author,

It is hard to believe that it has been over 36 years since Apollo-17 left the moon, and the last humans walked on its surface. It is difficult to look back at the intervening years and not dwell upon the opportunities that were lost in setting up the first human colonies there. Back then, there was a huge public outcry against the perceived wastefulness of sending humans to the moon, when so many humans remained to be fed here on Earth.

Now, of course, the human population has nearly doubled, (3.8 billion in 1972 to 6.7 billion). Despite the trillions of dollars that have been spent to relieve poverty, it is still with us. The 0.7% of our federal budget that we are allowed to spend on space exploration ($17 billion in 2007) is completely overshadowed by other huge purchases that we seldom complain about (for instance, bottled water $12 billion and pet food $17 billion), and the nearly $1 trillion now deeded to 'bail-out' our economy.

This math guide celebrates the impending return to the moon by NASA with the launch of the Lunar Reconnaissance Orbiter (LRO). It is the first mission in NASA's plan to return to the moon, and then to travel to Mars and beyond. LRO will launch sometime in 2009. Its main goals are to search for safe landing sites, locate potential resources for creating fuels and food such as water, and measure the radiation environment.

I hope you will enjoy this sample of problems, and help your students capture some of the excitement of exploring the universe through mathematics!

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