Compound Interest

<u>How it works:</u> Suppose this year I put $100.00 in the bank. The bank invests this money and at the end of the year gives me $4.00 back in addition to what I gave them. I now have $104.00. My initial $100.00 increased in value by 100% x ($104.00 - $100.00)/ $100.00) = 4%. Suppose I gave all of this back to the bank and they reinvested in again. At the end of the second year they have me another 4% increase. How much money do I now have? I get back an additional 4%, but this time it is 4% of $104.00 which is $104.00 x 0.04 = $4.16. Another way to write this after the second year is:

$$\$100.00 \times (1.04) \times (1.04) = \$108.16.$$

After 6 years, at a gain of 4% each year, my original $100.00 is now worth:

$$\$100.00 \times (1.04) \times (1.04) \times (1.04) \times (1.04) \times (1.04) \times (1.04) = \$126.53$$

<u>Do you see the pattern?</u> The basic formula that lets you calculate this 'compound interest' easily is:

$$F = B \times (1 + P/100)^{T}$$

where :
B = the starting amount, P= the annual percentage increase, T = number of investment years.

<u>Question:</u> In the formula, why did we divide the interest percentage by 100 and then add it to 1?

<u>Problem 1:</u> The US Space program invested $26 billion to build the Apollo Program to send 7 missions to land on the Moon.

A) What was the average cost for each Apollo mission?

B) You have probably heard your parents complain that 'prices have sure gone up this year!'. This is because, each year, the price for food, gasoline, and other things you buy as a family have been increasing each year by about 3% .This is called Inflation. It means that this year you have to pay $1.03 for something you bought for $1.00 last year. Since the last moon landing in 1972, inflation has averaged about 4% each year. From you answer to A), how much would it cost to do the same Apollo moon landing in 2007?

<u>Problem 2:</u> A NASA satellite program was originally supposed to cost $250 million when it started in 2000. Because of delays in approvals by Congress and NASA, the program didn't get started until 2005. If the inflation rate was 5% per year, A) how much more did the mission cost in 2005 because of the delays? B) Was it a good idea to delay the mission to save money in 2000?

<u>Problem 3:</u> A scientist began his career with a salary of $40,000 in 1980, and by 2000 this had grown to $100,000. A) What was his annual salary gain each year? B) If the annual inflation rate was 3%, why do you think that his salary gain was faster than inflation during this time?

Do you see the pattern? Each year you invest the money, you multiply what you started with the year before by 1.04.

$$F = B \times (1 + P/100)^{T}$$

Question: In the formula, why did we divide the interest percentage by 100 and then add it to 1? Because if each year you are increasing what you started with by 4%, you will have 4% more at the end of the year, so you have to write this as 1 + 4/100 = 1.04 to multiply it by the amount you started with.

Problem 1: The US Space program invested \$26 billion to build the Apollo Program to send 7 missions to land on the Moon. A) What was the average cost for each Apollo mission?

Answer : \$26 billion/7 = \$3.7 billion.

B) Answer: The number of years is 2007-1972 = 35 years. Using the formula, and a calculator:

$$F = \$3.7 \text{ billion } \times (1 + 4/100)^{35} = \$3.7 \text{ billion } \times (1.04)^{35} = \$14.6 \text{ billion.}$$

Problem 2: A) Answer: The delay was 5 years, so

$$F = \$250 \text{ million } \times (1 + 5/100)^{5} = \$250 \text{ million } \times (1.28) = \$319 \text{ million}$$

The mission cost \$69 million more because of the 5-year delay.

B) No, because you can't save money starting an expensive mission at a later time. Because of inflation, missions always cost more when they take longer to start, or when they take longer to finish.

Problem 3: A scientist began his career with a salary of \$40,000 in 1980, and by 2000 this had grown to \$100,000. A) What was his annual salary gain each year? Answer A) The salary grew for 20 years, so using the formula and a calculator, solve for X the annual growth:

$$\$100{,}000 = \$40{,}000 \times (X)^{20} \qquad X = (100{,}000/40{,}000)^{1/20} \qquad X = 1.047$$

So his salary grew by about 4.7% each year, which is a bit faster than inflation.

B) If the inflation rate was 3%, why do you think that his salary gain was faster than inflation during this time? Answer: His salary grew faster than inflation because his employers valued his scientific research and gave him average raises of 1.5% over inflation each year!